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ANALYSIS OF A MULTI-DEGREE OF FREEDOM  
VIBRATING SYSTEM WITH VISCOUS DAMPING  
USING THE DIGITAL COMPUTER

THOMAS J. MIKLOS











ANALYSIS OF A MULTI-DEGREE OF FREEDOM  
VIBRATING SYSTEM WITH VISCOUS DAMPING  
USING THE DIGITAL COMPUTER

\* \* \* \* \*

Thomas J. Miklos





ANALYSIS OF A MULTI-DEGREE OF FREEDOM  
VIBRATING SYSTEM WITH VISCOUS DAMPING  
USING THE DIGITAL COMPUTER

by

Thomas J. Miklos  
//  
Lieutenant, United States Navy

Submitted in partial fulfillment of  
the requirements for the degree of

MASTER OF SCIENCE  
IN  
MECHANICAL ENGINEERING

United States Naval Postgraduate School  
Monterey, California

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**ANALYSIS OF A MULTI-DEGREE OF FREEDOM  
VIBRATING SYSTEM WITH VISCOUS DAMPING  
USING THE DIGITAL COMPUTER**

by

**Thomas J. Miklos**

**This work is accepted as fulfilling  
the thesis requirements for the degree of  
MASTER OF SCIENCE  
IN  
MECHANICAL ENGINEERING  
from the  
United States Naval Postgraduate School**



## ABSTRACT

The general solution of the behavior of a viscously damped vibrating system having several degrees of freedom, as developed in the literature via matrix methods, is treated in detail here and made the basis of a digital computer program which is capable of determining the natural frequencies, the mode shapes, and the displacements of each mass as functions of time. This program, which is written in FORTRAN 60 language for the Control Data Corporation 1604 computer, affords several output options. It does not treat cases of supercritical damping or cases in which two or more natural frequencies are the same.



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## Nomenclature

|           |  |
|-----------|--|
| A         | independent variable in reduced system         |
| A'        | real part of a complex matrix                  |
| B         | independent variable in reduced system         |
| B'        | imaginary part of a complex matrix             |
| C         | damping matrix                                 |
| D         | driving force amplitude                        |
| DI        | imaginary component of driving force amplitude |
| DR        | real component of driving force amplitude      |
| F(t)      | forcing function                               |
| G         | column of constant coefficients                |
| I         | identity matrix                                |
| K         | stiffness matrix                               |
| M         | mass or inertia matrix                         |
| N         | degrees of freedom                             |
| O         | null matrix                                    |
| p         | eigenvalue                                     |
| R         | independent variable in reduced system         |
| r,s       | subscripts designating different modes         |
| SS        | steady state solution                          |
| TS        | transient solution                             |
| U         | eigenvector                                    |
| V         | real component of eigenvector                  |
| W         | imaginary component of eigenvector             |
| X         | generalized displacement, a column vector      |
| $\dot{X}$ | generalized velocity                           |



|            |                                      |
|------------|--------------------------------------|
| $\ddot{x}$ | generalized acceleration             |
| $Y$        | dependent variable in reduced system |
| $Z$        | dependent variable in reduced system |
| $\alpha$   | decrement factor ( $\zeta\omega_n$ ) |
| $\beta$    | damped natural frequency             |
| $\Phi$     | dependent variable in reduced system |
| $\omega$   | excitation frequency                 |
| $\omega_r$ | natural frequency of rth mode        |



## CHAPTER I

### INTRODUCTION

1.1 General Remarks. The analysis of a subcritically damped multiple degree of freedom vibrating system necessitates obtaining the solution of a complex eigenvalue problem to determine the natural frequencies and mode shapes of the system. Although the analysis presented in the literature for systems with two degrees of freedom may be extended to systems with more than two degrees of freedom, manual calculations are too laborious to be practical. Therefore the natural frequencies are usually found by ignoring the presence of the damping and solving the resulting real eigenvalue problem. This simplifies the problem considerably and provides a good approximation provided the damping is light. Another technique employed is to solve the real eigenvalue problem and then account for the damping using perturbation techniques. However, even in the absence of damping, systems involving more than two degrees of freedom usually require iteration or trial and error techniques (such as the Stodola or Holzer methods) to obtain the mode shapes.

The advent of the electronic digital computer has eliminated the necessity of ignoring the damping component and increased the size of the system for which solutions can be obtained. Although not entirely devoid of error the digital computer is highly reliable and its speed of operation has made it an invaluable tool in engineering analysis and design.



1.2 Method of Analysis. The multiple degree of freedom damped vibrating system is described by a set of N linear second order differential equations, where N denotes the degrees of freedom involved. Utilizing matrix analysis and generalized coordinates the system is described by

$$M\ddot{X} + C\dot{X} + KX = F(t)$$

which is the same form as the single degree of freedom system. However, M, C, and K now represent square matrices and X,  $\dot{X}$ ,  $\ddot{X}$ , and F(t) are column vectors. The analysis presented in what follows is performed using matrix technique because of the compactness of the notation and the ordered computational procedure which is ideally suited for digital computer programming.

The mathematical analysis is demonstrated and equations are derived which describe the time behavior of free and forced vibrating systems. Finally a digital computer program is presented which performs the operations indicated in the mathematical analysis to determine the natural frequencies, mode shapes, and time behavior of the damped vibrating system.





## CHAPTER II

### MATHEMATICAL ANALYSIS

2.1 Eigenvalues and Eigenvectors. The viscously damped vibrating system with one degree of freedom is described by a linear, second order differential equation.

$$M\ddot{X} + C\dot{X} + KX = F(t)$$

M, C, and K represent the mass, damping and stiffness of the system, and F(t) the forcing function. A system with many degrees of freedom is described by a set of second order differential equations similar to the case of the single degree of freedom system. Using matrix notation the system is described for the case of free vibration by

$$(1) \quad M\ddot{X} + C\dot{X} + KX = 0$$

where M, C, K, are now square matrices of order N, representing, as in the single degree case, the mass, damping and stiffness matrices of the system. It is always possible to write the equations so that these matrices are symmetric. N is the number of degrees of freedom. X,  $\dot{X}$ , and  $\ddot{X}$  are column vectors of order N representing displacement, velocity, and acceleration in generalized coordinates.

Premultiply equation (1) by  $M^{-1}$  (Capital letters will hereafter represent matrices and lower case letters scalar constants) to obtain;

$$(1a) \quad \ddot{X} + M^{-1}C\dot{X} + M^{-1}KX = 0$$

W. J. Duncan and A. R. Collar [1]<sup>\*</sup> have shown a simplified method of finding the eigenvalues and eigenvectors by writing the second order differential equation in reduced form as a first order differential equation. The

---

<sup>\*</sup>

Numbers in brackets refer to references listed in the bibliography page 23



reduced form is obtained by introducing velocity as a dependent variable.

Let

$$(2a) \quad Y = \begin{bmatrix} X \\ \dot{X} \end{bmatrix}$$

Here  $Y$  is a partitioned column matrix of order  $2N$  with the  $N$  displacement components in the upper half and  $N$  velocity components in the lower half.

Let

$$(2b) \quad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$$

Here  $0$  is an  $n$ th-order null matrix and  $I$  is an  $n$ th-order identity matrix.

The reduced equation then becomes

$$(3) \quad \dot{Y} = AY$$

A solution of a first order differential equation such as equation (3) may be taken as

$$(4) \quad Y = U e^{pt}$$

Substitution of the assumed solution into equation (3) gives

$$(5) \quad p e^{pt} U = A e^{pt} U$$

Dividing out the exponential term and rearranging, we have

$$(5a) \quad [A - pI]U = 0$$

To have a nontrivial solution, it then follows that

$$\det [A - pI] = 0$$

This is the characteristic matrix of the reduced system. The eigenvalues or roots of the characteristic equation are  $p_r$ ;  $r = 1, 2, \dots, 2N$  and the eigenvectors are  $U_r$ ;  $r = 1, 2, \dots, 2N$



where

$$u_r = \begin{bmatrix} u_{1,r} \\ u_{2,r} \\ \vdots \\ u_{2N,r} \end{bmatrix}$$

The first subscript indicates the position in the column and the second subscript corresponds to the  $r^{\text{th}}$  eigenvalue.

As long as the damping is less than critical in each mode, the reduced system will yield  $N$  complex conjugate pairs of eigenvalues and eigenvectors. The eigenvalues will be of the same form as the subcritically damped single degree of freedom system.

$$\rho_r = -\zeta_r \omega_r + j \omega_r \sqrt{1 - \zeta_r^2}$$

$$\bar{\rho}_r = -\zeta_r \omega_r - j \omega_r \sqrt{1 - \zeta_r^2}$$

The bar indicates the complex conjugate and the subscript indicates the mode, and where

$\zeta_r$  - damping ratio

$\omega_r$  - natural frequency

$\omega_r \sqrt{1 - \zeta_r^2}$  - damped natural frequency

$\zeta_r \omega_r$  - decrement factor

The corresponding eigenvectors are

$$u_r = v_r + j w_r$$

$$\bar{u}_r = v_r - j w_r$$

## 2.2 Homogeneous Equation.

**2.2.1 Orthogonality Relations.** In the case of the undamped vibrating system it is always possible to choose a set of coordinates in which the mass and stiffness matrices are uncoupled. However in the damped vibrating system, unless the damping matrix is proportional to either the mass or stiffness matrix, a set of coordinates which will uncouple the equations



of motion cannot be found without a knowledge of the eigenvalues and eigenvectors of the system.

K. A. Foss has developed a set of orthogonality relations for the eigenvectors of a damped linear dynamic system from which coordinates which uncouple the equations of motion may be found. [2] The technique of reducing the second order differential equation to one of the first order is again employed, with slightly different notation.

Let

$$(6a) \quad R = \begin{bmatrix} O & M \\ M & C \end{bmatrix} \quad (6b) \quad B = \begin{bmatrix} -M & O \\ O & K \end{bmatrix} \quad (6c) \quad Z = \begin{bmatrix} \dot{X} \\ X \end{bmatrix}$$

Using the above notation, equation (1) becomes

$$(7) \quad R \dot{Z} + BZ = O$$

Furthermore let

$$X = U_r e^{p_r t}$$

$$\dot{X} = p_r U_r e^{p_r t}$$

Then

$$(8) \quad Z = \Phi_r e^{p_r t}$$

where  $\Phi_r = \begin{bmatrix} p_r U_r \\ U_r \end{bmatrix}$

Substituting equation (8) into equation (7) and dividing out the exponential factor we get,

$$(9) \quad p_r R \Phi_r + B \Phi_r = O$$

Since the eigenvalues and eigenvectors occur in complex conjugate pairs the complex conjugates of  $U_r$  and  $p_r$  also satisfy the homogeneous equation.





Denoting the complex conjugate by the subscript S, equation (9) may be written

$$(10) \quad P_S R \Phi_S + B \Phi_S = 0$$

Or since M, C, and K are symmetric, R and B must be symmetric, and equation (10) may be written

$$(10a) \quad P_S \Phi_S^T R + \Phi_S^T B = 0$$

Premultiply equation (9) by  $\Phi_S^T$ , postmultiply equation (10a) by  $\Phi_r$  and subtract (10a) from (9).

$$\begin{array}{rcl} P_r \Phi_S^T R \Phi_r + \Phi_S^T B \Phi_r & = & 0 \\ - P_S \Phi_S^T R \Phi_r + \Phi_S^T B \Phi_r & = & 0 \\ \hline (P_r - P_S) \Phi_S^T R \Phi_r & = & 0 \end{array}$$

Therefore unless  $r = s$  the orthogonality relations are

$$(11) \quad \Phi_S^T R \Phi_r = 0$$

$$(12) \quad \Phi_S^T B \Phi_r = 0$$

2.2.2. General solution of the homogeneous equation. In the general solution of equation (7),  $2N$  arbitrary constants of integration must be evaluated. The  $2N$  initial conditions of displacement and velocity are used to evaluate these constants. Assume a solution of the form

$$(13) \quad \mathbf{z}(t) = \sum_{r=1}^{2N} \Phi_r C_r e^{P_r t}$$

where  $C_r$  represents the  $2N$  constants to be determined.



The initial conditions may be expressed as a vector expansion of the natural modes

$$(14) \quad Z(0) = \sum_{r=1}^{2N} \Phi_r C_r$$

Premultiply both sides of equation (14) by  $\Phi_s^T R$

$$(15) \quad \Phi_s^T R Z(0) = \sum_{r=1}^{2N} \Phi_s^T R \Phi_r C_r$$

Using the orthogonality relation, equation (11), the summation simplifies and equation (15) becomes

$$(16) \quad \Phi_s^T R Z(0) = \Phi_s^T R \Phi_s C_s$$

Or

$$(17) \quad C_s = \frac{\Phi_s^T R Z(0)}{\Phi_s^T R \Phi_s}$$

The general solution is therefore found by substituting the value of the constants of integration into equation (13).

$$(18) \quad Z(t) = \sum_{r=1}^{2N} \frac{\Phi_r \Phi_r^T R}{\Phi_r^T R \Phi_r} Z(0) e^{P_r t}$$

It follows from an evaluation of equation (18) at time zero, that

$$\sum_{r=1}^{2N} \frac{\Phi_r \Phi_r^T R}{\Phi_r^T R \Phi_r} = I$$

Equation (18) may be written in the following partitioned form using the nth-order system notation.

$$(19) \quad \begin{bmatrix} \dot{X}(t) \\ X(t) \end{bmatrix} = \sum_{r=1}^{2N} \frac{\begin{bmatrix} P_r U_r U_r^T M \dot{X}(0) + P_r^2 U_r U_r^T M X(0) + P_r U_r U_r^T C X(0) \\ U_r U_r^T M \dot{X}(0) + P_r U_r U_r^T M X(0) + U_r U_r^T C X(0) \end{bmatrix}}{2 P_r U_r^T M U_r + U_r^T C U_r} e^{P_r t}$$

from which

$$(20) \quad X(t) = \sum_{r=1}^{2N} \frac{\begin{bmatrix} U_r U_r^T M \dot{X}(0) + P_r U_r U_r^T M X(0) + U_r U_r^T C X(0) \end{bmatrix}}{2 P_r U_r^T M U_r + U_r^T C U_r} e^{P_r t}$$



It should be noted again at this point that for the sub-critically damped system being considered,  $p_r$  and  $U_r$  are complex quantities and occur in complex conjugate pairs. The matrix operations indicated in equation (20) have been carried out by breaking up the complex quantities into real and imaginary parts and then recombining. The operations are relatively straightforward but lengthy, therefore only the final form will be presented.

$$(21) \quad X(t) = \sum_{r=1}^N \frac{2}{a_r^2 + b_r^2} \left\{ \left[ G_r M \dot{X}(0) + (-\alpha_r G_r M + \beta_r H_r M + G_r C) X(0) \right] \cos \beta_r t \right\} e^{-\alpha_r t} \\ + \sum_{r=1}^N \frac{2}{a_r^2 + b_r^2} \left\{ \left[ H_r M \dot{X}(0) + (-\alpha_r H_r M - \beta_r G_r M + H_r C) X(0) \right] \sin \beta_r t \right\} e^{-\alpha_r t}$$

Where:

$$p_r = -\alpha_r + j\beta_r \quad U_r = V_r + jW_r \\ a_r = -2\alpha_r (V_r^T M V_r - W_r^T M W_r) - 4\beta_r V_r^T M W_r + V_r^T C V_r - W_r^T C W_r \\ b_r = 2\beta_r (V_r^T M V_r - W_r^T M W_r) - 4\alpha_r V_r^T M W_r + 2 V_r^T C W_r \\ A_r = V_r V_r^T - W_r W_r^T \\ B_r = V_r W_r^T + W_r V_r^T \\ G_r = a_r A_r + b_r B_r \\ H_r = b_r A_r - a_r B_r$$

The factor of two in the above equation results from the reduction of the summation from  $2N$  to  $N$ . When the indicated operations of equation (20) are carried out from  $r = 1$  to  $r = 2N$  the imaginary components sum to zero; therefore all quantities in equation (21) are real. Equation (21) has been given by S. H. Crandall and R. B. McCalley in reference 3.



## 2.3 Non-Homogeneous Equation

2.3.1 Steady State Solution of the Non-Homogeneous Equation. Once again using the reduced form, the non-homogeneous equation becomes

$$(22) \quad R\dot{Z} + BZ = F(t)$$

Where

$$F(t) = \begin{bmatrix} 0 \\ f(t) \end{bmatrix}$$

In a linear nth-order differential equation the solution of the non-homogeneous equation may be found, once the solution of the homogeneous equation is known, by expanding  $Z$  in a modal series. [2,4]

Thus,

$$(23) \quad Z(t) = \sum_{r=1}^{2N} \Phi_r Y_r(t)$$

here  $Y_r(t)$  is a variable parameter.

Upon substitution of equation (23) in equation (22), we obtain

$$(24) \quad \sum_{r=1}^{2N} R \Phi_r \dot{Y}_r + B \Phi_r Y_r = F(t)$$

Premultiplication of equation (24) by  $\Phi_r^T$  and making use of the orthogonality relations, yields

$$(25) \quad \Phi_r^T R \Phi_r \dot{Y}_r + \Phi_r^T B \Phi_r Y_r = \Phi_r^T F(t)$$

Or

$$(25a) \quad R_n \dot{Y} - p_r R_n Y = F_n$$

Where

$$R_n = \Phi_r^T R \Phi_r$$

$$-p_r R_n = \Phi_r^T B \Phi_r$$

$$F_n = \Phi_r^T F(t)$$





Using the Laplace Transform technique, the solution of equation (25a) is easily determined.

Rearranging

$$(a) \quad \dot{Y} - p_r Y = \frac{F_n}{R_n}$$

The Laplace Transform is

$$(b) \quad sY(s) - p_r Y(s) = \frac{F_n(s)}{R_n}$$

Therefore

$$(c) \quad Y(s) = \frac{1}{R_n} \cdot \frac{F_n(s)}{(s - p_r)}$$

The inverse transform of (c) is

$$(26) \quad Y(t) = \frac{1}{R_n} \int_0^t e^{p_r(t-\tau)} F_n(\tau) d\tau$$

Since  $p_r = -\alpha_r + j\beta_r$

$$(26a) \quad Y(t) = \frac{1}{R_n} \int_0^t e^{-\alpha_r(t-\tau)} F_n(\tau) (\cos \beta_r(t-\tau) + j \sin \beta_r(t-\tau)) d\tau$$

Substitute (26a) into (23)

$$(27) \quad Z(t) = \sum_{r=1}^{2N} \frac{\Phi_r \Phi_r^T}{R_n} \int_0^t e^{-\alpha_r(t-\tau)} F(\tau) (\cos \beta_r(t-\tau) + j \sin \beta_r(t-\tau)) d\tau$$

As in the case of the homogeneous solution the reduced system quantities are replaced by their nth-order equivalence. The final form of the steady state solution then becomes

$$(28) \quad X(t) = \sum_{r=1}^N \frac{2G_r}{a_r^2 + b_r^2} \int_0^t f(\tau) e^{-\alpha_r(t-\tau)} \cos \beta_r(t-\tau) d\tau \\ + \sum_{r=1}^N \frac{2H_r}{a_r^2 + b_r^2} \int_0^t f(\tau) e^{-\alpha_r(t-\tau)} \sin \beta_r(t-\tau) d\tau$$

The notation in equation (28) is the same as that used in the homogeneous solution.



2.3.2 Steady State Solution with Sinusoidal Excitation. A special, but very common problem is the case in which the forcing function,  $f(t)$ , is sinusoidal. Expressing the forcing function as

$$(29) \quad f(t) = \mathcal{R}(De^{j\omega t})$$

where  $\mathcal{R}$  - indicates "the real part of"

$D$  - column of driving force amplitudes, possibly complex to admit phasing

$\omega$  - excitation frequency

The non-homogeneous equation therefore becomes

$$(30) \quad M\ddot{x} + C\dot{x} + Kx = \mathcal{R}(De^{j\omega t})$$

Assume a solution of the form

$$(31) \quad x(t) = \mathcal{R}(Ge^{j\omega t})$$

where  $G$  is a column of undetermined constant coefficients. Substitute the assumed solution in equation (30) and divide through by the exponential factor to obtain

$$(32) \quad -\omega^2 MG + j\omega CG + KG = D$$

Or

$$(33) \quad G = [K - \omega^2 M + j\omega C]^{-1} D$$

Therefore the steady state solution for the case of sinusoidal excitation is

$$(34) \quad x(t) = \mathcal{R}\left\{[K - \omega^2 M + j\omega C]^{-1} D e^{j\omega t}\right\}$$



## 2.4 Transient Plus Steady State Solution.

2.4.1. Initial Conditions. The total solution is given by

$$(35) \quad X(t) = TS + SS$$

where TS is the so called transient solution and SS the so called steady state solution. However in the total solution the initial conditions of displacement and velocity in the transient solution must account for the initial values in the steady state solution. This is accomplished as follows. In the case of sinusoidal excitation the steady state solution was found to be

$$(34) \quad X(t) = \mathcal{R} \left\{ \left[ K - \omega^2 M + j\omega C \right]^{-1} D e^{j\omega t} \right\}$$

To allow for phase differences in the exciting forces on the masses, let  $D$  be given by

$$(36) \quad D = DR + jDI$$

and let the inverse of  $[K - \omega^2 M + j\omega C]$  be denoted by

$$(37) \quad A' + jB'$$

Then since  $e^{j\omega t} = \cos \omega t + j \sin \omega t$ , equation (34)

becomes

$$(38) \quad X(t) = \mathcal{R} \left\{ \left[ A' + jB' \right] \left[ DR + jDI \right] (\cos \omega t + j \sin \omega t) \right\}$$

Carrying out the indicated multiplications, we obtain

$$(39) \quad X(t) = \left\{ (A'DR - B'DI) \cos \omega t - (B'DR + A'DI) \sin \omega t \right\}$$



and the velocity is given by

$$(40) \quad \dot{X}(t) = \left\{ -\omega (A'DR - B'DI) \sin \omega t - \omega (B'DR + A'DI) \cos \omega t \right\}$$

Therefore the initial values of displacement and velocity of the steady state solution are obtained by evaluating equations (38) and (39) at  $t=0$

$$(41) \quad X_{ss}(0) = A'DR - B'DI$$

$$(42) \quad \dot{X}_{ss}(0) = -\omega (B'DR + A'DI)$$

The initial conditions used in determining the time behavior of the transient portion for the total solution therefore become

$$(43) \quad X_c(0) = X(0) - (A'DR - B'DI)$$

$$(44) \quad \dot{X}_c(0) = \dot{X}(0) + \omega (B'DR + A'DI)$$

The complete time-solution is thus obtained as indicated in equation (35) where the transient solution is found by using the modified initial conditions of equations (43) and (44).





## CHAPTER III

### PROGRAM DESCRIPTION

3.1 General Remarks. A digital computer program "PROGRAM VIBSYS" is presented which performs the mathematical operations of the equations developed in the previous chapter. The program is coded in Fortran 60 programming language, [5,6] specifically for the Control Data Corporation 1604 computer. Although the Fortran 60 language is applicable to most large digital computers there are minor variations which are peculiar to the specific system in use. The Fortran 60 language does not permit automatic operations with complex numbers; therefore all operations involving complex numbers are accomplished by operating on the real and imaginary parts separately and then recombining.

The eigenvalues and eigenvectors of the reduced system are found by a matrix iteration scheme which utilizes the direct and inverse power methods and matrix deflation. A mathematical subroutine MATSUB is used to carry out these operations. [7,8] As presented, a maximum of twenty iterations will be performed using the direct power method and then a maximum of twenty using the inverse power method. If convergence is not reached with the inverse power method a print out to this effect will be executed.

Subroutine "INVERT" is used for matrix inversion.\* "INVERT" uses the Gaussian elimination and pivotal techniques. Inversion of the  $[K - \omega^2 M + j\omega C]$  matrix which contains complex elements is achieved in

---

\*

Subroutine INVERT is a library subroutine of the Naval Postgraduate School and is designated locally as F1 NPS INVERT.



the following manner. Let  $[A + jB]$  be the matrix to be inverted and  $[C + jD]$  the inverse to be determined. Then by definition

$$[A + jB][C + jD] = I$$

The above matrix equation leads to two simultaneous equations with two unknowns, C and D.

$$(a) \quad AC - BD = I$$

$$(b) \quad BC + AD = 0$$

Solving first for C

$$C = [A + BA^{-1}B]^{-1}$$

and then for D

$$D = -A^{-1}BC$$

Thus the complex inversion is reduced to multiplications, additions, and inversions, of matrices of real numbers.

Flexibility and utility are the principal aims of the program. Usage requires a knowledge of the mass, stiffness, and damping matrices. Although various output options are available which require additional input data, the three above mentioned matrices are all that are necessary to determine the eigenvalues and eigenvectors of the reduced system and the natural frequencies and mode shapes of the original system.

**3.2 Program Options.** In addition to finding the natural frequencies and mode shapes of the system, five options are available which describe the time behavior of the system under conditions of free and forced vibration.

(a) Option 1. The time solution of the free vibration problem is obtained in general form and no additional input data is required for execution of this option. In section 2.2.2 the general solution of the homogeneous equation was developed and is repeated here for convenience.



$$(21) \quad X(t) = \sum_{r=1}^N \frac{2}{a_r^2 + b_r^2} \left\{ \left[ G_r M \dot{X}(0) + (-\alpha_r G_r M + \beta_r H_r M + G_r C) X(0) \right] e^{-\alpha_r t} \cos \beta_r t \right. \\ \left. + \sum_{r=1}^N \frac{2}{a_r^2 + b_r^2} \left\{ \left[ H_r M \dot{X}(0) + (-\alpha_r H_r M - \beta_r G_r M + H_r C) X(0) \right] e^{-\alpha_r t} \sin \beta_r t \right. \right.$$

The output of option 1 consists of the four coefficient matrices of the  $\dot{X}(0) \cos \beta_r t$ ,  $X(0) \cos \beta_r t$ ,  $\dot{X}(0) \sin \beta_r t$  and  $X(0) \sin \beta_r t$  terms. Therefore, the output of option 1 will consist of  $4N$  square matrices.

(b) Option 2. The execution of option 2 requires as additional input data the values of the initial displacement, and initial velocity vectors. The product of the coefficient matrices of option 1 and the initial displacement vector or initial velocity vector, as appropriate, is performed to obtain a coefficient column for the cosine and sine terms.

(c) Option 4.\* The general steady state solution of the forced vibration problem is provided by option 4. The general solution with a forcing function  $f(t)$  was developed in section 2.3.1 and found to be

$$(28) \quad X(t) = \sum_{r=1}^N \frac{2 G_r}{a_r^2 + b_r^2} \int_0^t f(\tau) e^{-\alpha_r(t-\tau)} \cos \beta_r(t-\tau) d\tau \\ + \sum_{r=1}^N \frac{2 H_r}{a_r^2 + b_r^2} \int_0^t f(\tau) e^{-\alpha_r(t-\tau)} \sin \beta_r(t-\tau) d\tau$$

In this case the coefficient matrices of the convolution integral are evaluated. There will be  $2N$  square matrices,  $N$  corresponding to the coefficient of the integral involving the cosine term and  $N$  coefficient matrices of the integral involving the sine term.

(d) Option 5. The steady state solution of the special case of forced vibration with sinusoidal excitation is determined in option 5.

---

\*

Option 3 is described in subsection (e), following.





In section 2.4.1 the expression for the time behavior was found to be

$$(39) \quad X(t) = (\dot{A}DR - B'DI) \cos \omega t - (B'DR + \dot{A}DI) \sin \omega t$$

Additional input data required for the execution of this option include the driving force amplitude and excitation frequency. The output consists of the coefficient column vectors of the cosine and sine terms.

(e) Option 3. A plot of displacement versus time is made for each mass, for one of the three following cases.

1. Free Vibration
2. Steady state vibration with sinusoidal excitation
3. Transient plus steady state with sinusoidal excitation

Case 1 is obtained when the initial conditions of displacement and velocity are given as input data and the driving force amplitudes and excitation frequency are zero.

Case 2 is obtained when the driving force amplitudes and excitation frequency are given and the initial conditions of displacement and velocity are zero.

Case 3 is obtained when the initial conditions, driving force amplitudes and excitation frequency are all given.

Any combination of the options may be obtained with one set of input data. Input data format and output control is described in detail in Appendix B.

3.3 Accuracy of Method. The accuracy of the solution is contingent on the accuracy with which the eigenvalues and eigenvectors are determined and the effect of computer roundoff error. Internal checks have been provided so that the integrity of the solution may readily be evaluated.





3.3.1 Internal Checks. Prior to the determination of the eigenvalues and eigenvectors the trace and determinate of the matrix, A, is calculated. A check is therefore readily available since the trace of the matrix A is equal to the sum of the eigenvalues of the characteristic matrix of A and the determinant is equal to the product of the eigenvalues. The trace and determinant of A and the sum and product of the eigenvalues are included as standard output in Program VIBSYS.

Additional checks are inherent in the solution. In Option 1 the summation of the coefficient matrices of the  $x(0) \cos \theta_r t$  terms must sum to the identity matrix, since the evaluation of  $x(t)$  at time zero must equal the initial conditions of displacement. This is easily seen by referring to equation 21 and considering the initial values of velocity to be zero. Similarly, in Option 2 the sum of the coefficient columns of the cosine terms must equal the initial conditions of displacement.

In the steady state solution with sinusoidal excitation, the inversion of the complex matrix presents one of the greatest possible sources of error. Therefore when Option 5 is executed, the output includes the real and imaginary parts of the inverse and the product of the original matrix and its inverse. This product should, of course, be the identity matrix.

3.3.2 External Checks. In order to determine the reliability of Program VIBSYS, sample problems of two degrees of freedom for which solutions were available in the literature were programmed. Correlation of results was excellent. No suitable sample problems involving more than two degrees of freedom were found in the literature. Therefore, in order to extend the range of testing, solutions of undamped systems were compared with program solutions of similar systems with negligible damping.



The results were as anticipated; as the damping was decreased the natural frequencies approached those of the undamped system. The results of a sample problem are demonstrated in Appendix E.

3.4 Program Limitations. Although the digital computer extends the size of system for which solutions are obtainable, the ultimate size is dictated by the storage capacity of the computer. The CDC 1604 computer has approximately 32,000 storage locations; however, it is easily seen that this may be rapidly exhausted. Program VIBSYS is limited to a system with 10 degrees of freedom.

The basic principle in the development of the general equations is the orthogonality relations of the eigenvectors and the parameters of the system. Therefore each eigenvalue must have associated with it a unique eigenvector. This requirement is not fulfilled in the case of a system having two equal eigenvalues, and therefore two natural frequencies of equal magnitude. In such a case VIBSYS determines the frequencies and the program terminates at that point.

If the damping is critical in any part of the system an attempt will be made to obtain the eigenvalues and eigenvectors. However, if they are found, they will not occur in the complex conjugate pairs and the program will be terminated.



## CHAPTER IV

### CONCLUSIONS AND RECOMMENDATIONS

Program VIBSYS provides an accurate method for determining the natural frequencies, mode shapes, and time behavior of a subcritically damped dynamic system. The compiling time for the program is 4 minutes and 30 seconds, while the running time for a system with four degrees of freedom is approximately one minute. (Running time will depend on the number of iterations required to obtain the eigenvalues and eigenvectors.) Therefore it is more efficient, with respect to computer time, to make multiple runs.

Comparison of natural frequencies of undamped and lightly damped systems has shown that the undamped approximation is very good for damping ratios of less than 0.01. For systems tested, up to four degrees of freedom, the natural frequencies of the damped and undamped systems did not differ in the first three significant figures.

The program may be extended in a variety of possible ways. The general nature of Program VIBSYS requires liberal use of computer storage space. The size of the system may be extended by segmenting the program into separate programs for handling specific problems such as, free vibration, forced vibration with a general forcing function, or forced vibration with sinusoidal excitation. Furthermore, with minor modifications the present program may be terminated upon finding the natural frequencies and mode shapes and used as a separate program to obtain input data for specialized programs.

Program VIBSYS can be augmented and made more useful by eliminating the restrictions which cause the program to be terminated if either (a)



the system has two eigenvalues of equal magnitude, or (b) one of the modes has aperiodic free motion, that is a purely real eigenvalue.





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## APPENDIX A

### PROGRAM STRUCTURE

A.1 General Remarks. Program VIBSYS is composed of a main body which is divided into five sections, and two subroutines, INVERT and MATSUB. Function subroutines used include the sine, cosine, exponential, square root, and absolute value functions. These function subroutines are called from the Fortran library tape. Subroutine draw is also available on the Fortran library type and is programmed specifically for the CDC 1604 computer for use with the CALCOMP plotter.

Subroutine MATSUB, which is available as a CO-OP Library mathematical subroutine, required minor modifications in order to retain the eigenvalues and eigenvectors in storage for use in the main body of the program. The flow chart provided with MATSUB in the CO-OP Library was found to be inadequate for understanding. In order to perform the necessary modifications a revised flow chart was made up and included in Appendix C.

A.2 Main Body. The main body consists of five sections with functions as listed below

(a) Input - In addition to allocation of storage spaces for dimensioned arrays and setting up a control for reading input data, the input section contains a control sequence for MATSUB and curve labeling.

(b) Natural frequencies and mode shapes - The reduced characteristic matrix is constructed and the eigenvalues and eigenvectors are determined. The natural frequencies and mode shapes are then found.

(c) Forced Vibration - The coefficient column vectors of the sine and cosine terms of option 5 are calculated.

(d) Free Vibration - The outputs of options 1, 2, and 3 are formulated.



(e) Forced Vibration with General Excitation - The coefficient matrices of option 4 are calculated.

### A.3 Subroutines.

(a) MATSUB - Evaluates the eigenvalues and eigenvectors of the reduced system.

(b) INVERT - Inverts a real matrix.



# PROGRAM NOTATION

|       |   |   |
|-------|---|---|
| AB    | - | coefficient column vector of $\cos \beta_r t$ term, output of option 2  |
| ABC   | - | coefficient column vector of $\cos \beta_r t$ term when $X(0)$ , and $\dot{X}(0)$ are modified to obtain total solution |
| ALIS  | - | control parameter for MATSUB  |
| ALRS  | - | control parameter for MATSUB  |
| AM    | - | inverse of mass matrix  |
| AMD   | - | product of inverse of mass matrix and damping matrix  |
| AMM   | - | mass matrix lb-sec <sup>2</sup> /in   |
| AMS   | - | product of inverse of mass matrix and stiffness matrix  |
| ARN   | - | $-2 \alpha_r (V_r^T M V_r - W_r^T M W_r) - 4 \beta_r [V_r^T M W_r + V_r^T C V_r - W_r^T C W_r]$                         |
| ARY   | - | imaginary part of identity matrix   |
| BRN   | - | $2 \beta_r (V_r^T M V_r - W_r^T M W_r) - 4 \alpha_r V_r^T M W_r + 2 V_r^T C V_r$  |
| CD    | - | coefficient column vector of $\sin \beta_r t$ term, output of option 2  |
| CDC   | - | coefficient column vector of $\sin \beta_r t$ term when $X(0)$ and $\dot{X}(0)$ are modified to obtain total solution   |
| CPD   | - | coefficient column of $\sin \omega t$ , output of option 5  |
| DI    | - | imaginary part of driving force amplitude   |
| DISPI | - | imaginary part of modal matrix  |
| DISPR | - | real part of modal matrix   |
| DM    | - | damping matrix lb-sec/in  |
| DR    | - | real part of driving force amplitude  |
| EP1   | - | iteration parameter for MATSUB  |
| EP2   | - | iteration parameter for MATSUB  |





|        |   |  |
|--------|---|--|
| GBI    | - | iteration parameter for MATSUB   |
| GBR    | - | iteration parameter for MATSUB   |
| GRM    | - | coefficient matrix of $\dot{X}(0) \cos \beta_r t$ , output of option 1 |
| HRM    | - | coefficient matrix of $\dot{X}(0) \sin \beta_r t$ , output of option 1 |
| IC     | - | control parameter for VIBSYS   |
| IDET   | - | control parameter for MATSUB   |
| IEG    | - | control parameter for MATSUB   |
| IFC    | - | control parameter for VIBSYS   |
| ITITLE | - | title for graphical output   |
| IVEC   | - | control parameter for MATSUB   |
| MIT    | - | controls number of iterations in MATSUB, power method                  |
| MIT5   | - | controls number of iterations in MATSUB inverse power method           |
| MP1    | - | option 1 control   |
| MP2    | - | option 2 control   |
| MP3    | - | option 3 control   |
| MP4    | - | option 4 control   |
| MP5    | - | option 5 control   |
| N      | - | order of system  |
| RIN    | - | real part of identity matrix   |
| RMI    | - | coefficient column vector of $\cos \omega t$ term, output of option 5  |
| RPI    | - | real part of the inverse of $[K - \omega^2 M + j \omega C]$            |
| S      | - | stiffness matrix lb/in   |
| SPEC   | - | spectral matrix  |
| STEP   | - | step size for graphical output   |
| TIPI   | - | imaginary part of inverse of $K - \omega^2 M + j \omega C$             |



|      |   |   |
|------|---|---|
| UI   | - | matrix of reduced system, imaginary part  |
| UR   | - | matrix of reduced system, real part   |
| VALI | - | imaginary part of eigenvalue of reduced system ( $\beta_r$ )  |
| VALR | - | real part of eigenvalue of reduced system ( $\alpha_r$ )  |
| VCO  | - | initial velocity vector modified to account for initial velocity of steady state solution               |
| VCV  | - | $V_r^T C V_r$   |
| VCW  | - | $V_r^T C W_r$   |
| VECI | - | imaginary part of eigenvector of reduced system ( $V_r$ )   |
| VECR | - | real part of eigenvector of reduced system ( $W_r$ )  |
| VO   | - | initial velocity vector   |
| VMV  | - | $V_r^T M V_r$   |
| VMW  | - | $V_r^T M W_r$   |
| WCW  | - | $W_r^T C W_r$   |
| WDM  | - | imaginary part of $[K - \omega^2 M + j \omega C]$   |
| WMK  | - | real part of $[K - \omega^2 M + j \omega C]$  |
| WMW  | - | $W_r^T M W_r$   |
| WO   | - | excitation frequency  |
| X    | - | time coordinate for graphs  |
| XCO  | - | initial condition of displacement modified to account for initial displacement of steady state solution |
| XO   | - | initial displacement vector   |
| XOC  | - | coefficient matrix of $X(0) \cos \beta_r t$ term, output of option 1                                    |
| XOS  | - | coefficient matrix of $X(0) \sin \beta_r t$ term, output of option 2                                    |
| Y    | - | ordinate of graphical output  |

The above notation lists the principal array and parameter names used in the main body of the program. Array names not listed are used for intermediate operations. Where appropriate the names are associated with the



symbols used in the mathematical analysis.



## APPENDIX B

### INSTRUCTIONS FOR USE OF PROGRAM

B.1 Purpose. The purpose of Program VIBSYS is to determine the natural frequencies, mode shapes, and time behavior of a subcritically damped, linear dynamic system with N degrees of freedom. ( $N \leq 10$ ) The mass, damping and stiffness matrices of the system must be known.

B.2 Input Data. A blank card must follow the last end card of the program deck. The data cards then follow in the order and format listed below. The first data card uses the input format 9I5 and is the control card for the program. The nine fields are designated as follows:

1. N = the order of the system
2. MP1 = 1, Option 1 executed  
0, Omit
3. MP2 = 1, Option 2 executed  
0, Omit
4. MP3 = 1, Option 3 executed  
0, Omit
5. MP4 = 1, Option 4 executed  
0, Omit
6. MP5 = 1, Option 5 executed  
0, Omit
7. IC = 1, Initial conditions given  
0, Initial conditions are zero
8. IFC = 1, Amplitude and frequency of excitation given  
0, Omit
9. NPTS Number of points to be calculated for graphical output

All nine fields must be right justified.





The control card is followed by the mass, damping, and stiffness matrices respectively. Each matrix is read in rowwise using input format 8F10.3\* with each row starting on a new card for systems with  $N \leq 8$ . For systems with  $N > 8$  the elements are read in rowwise in a sequential manner, that is the first element of the second row should appear in the field adjacent to the last element of the first row. In this case each matrix must start on a new card.

The initial displacement vector, initial velocity vector, real part of the driving force amplitude vector, and imaginary part of the driving force amplitude vector, follow, in that order, using format 8F10.3.\* Each vector must start on a new card.

The final data card uses a 2F10.3\* format. The first field contains the excitation frequency and the second the step size (i.e., time increment) for the graphical output option.

Blank cards must be used for initial conditions of displacement and velocity, real and imaginary parts of forcing function amplitudes and final data card when these values are zero. Multiple runs may be processed by placing additional data decks behind the first. The program is terminated by placing a blank card behind the last data card.

---

\*

Should this format present undesirable restrictions on the size of the elements of the matrix the so called "E" field may be used by changing card number 32. The "E" field was avoided since it is more susceptible to error by the user.



B.3 Deck Assembly. The first card of the program deck is a job card. The statement "Use scratch tape" must be included in addition to the required job card information.

JOB CARD

PROGRAM VIBSYS

END

SUBROUTINE INVERT

END

SUBROUTINE MATSUB

END

END

BLANK CARD

DATA CARDS

BLANK CARD

B.4 Cautions to User. The curves drawn in the graphical output option are straight line approximations between computed values. The step size must therefore be chosen appropriately in order to obtain a smooth curve. Since the maximum number of points permitted by subroutine DRAW is 900, it



may not be possible to see the transient phase completely die out.

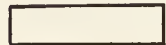
If difficulty is encountered in obtaining the eigenvalues and eigenvectors, the control parameters of MATSUB may be altered to enhance convergence. The value of MIT and MITS, card numbers 20 and 21, control the maximum number of iterations to be performed in the power method and inverse power method respectively. ALRS and ALIS, card numbers 78 and 79, represent a complex parameter that may be called the "origin". MATSUB will usually converge on the eigenvalue most distant from the "origin". ALRS = 1.0 and ALIS = 0.1 in Program VIBSYS. The convergence criteria for the power method is set by the value of EP1, card number 85 while the inverse power method is determined by EP2 card number 86.  $EP1 = 10^{-4}$ , and  $EP2 = 10^{-14}$  in the present version. Furthermore, if IEG, card number 76, is set equal to one, the eigenvalue iterants will be printed out and more suitable values of ALRS and ALIS may possibly be found by inspection.




## APPENDIX C


### FLOW CHARTS

The symbols used in the flow charts are defined as follows:

 - arithmetic, read or print operations

 - connectors

 - decision

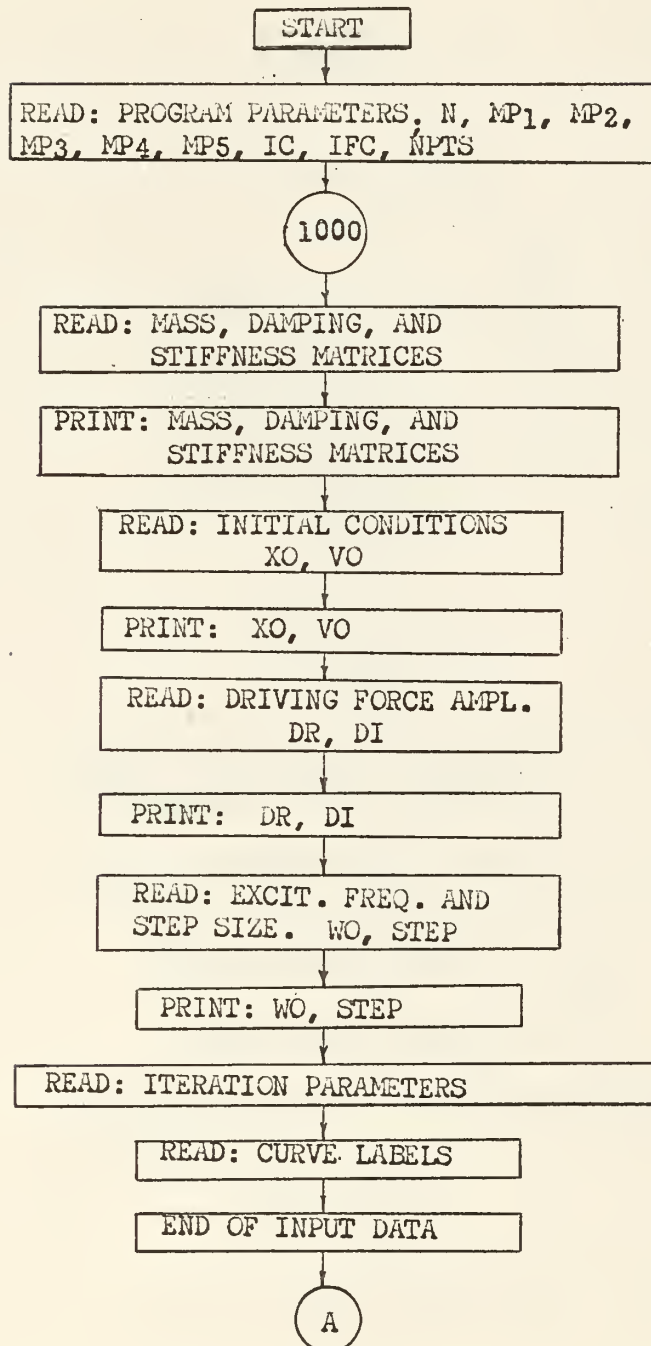
 - call subroutine indicated

The notation used in the flow chart for MATSUB corresponds to that used in the CO OP Library version, reference 7.

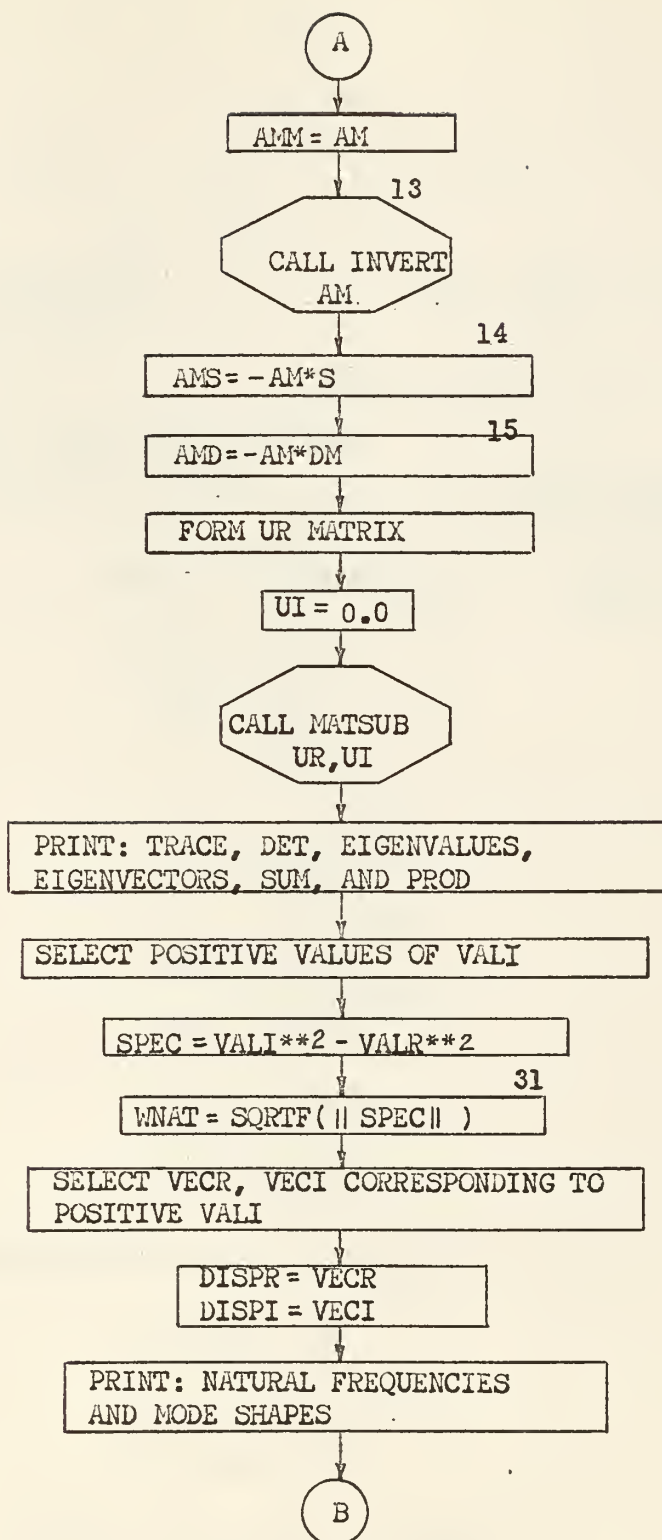




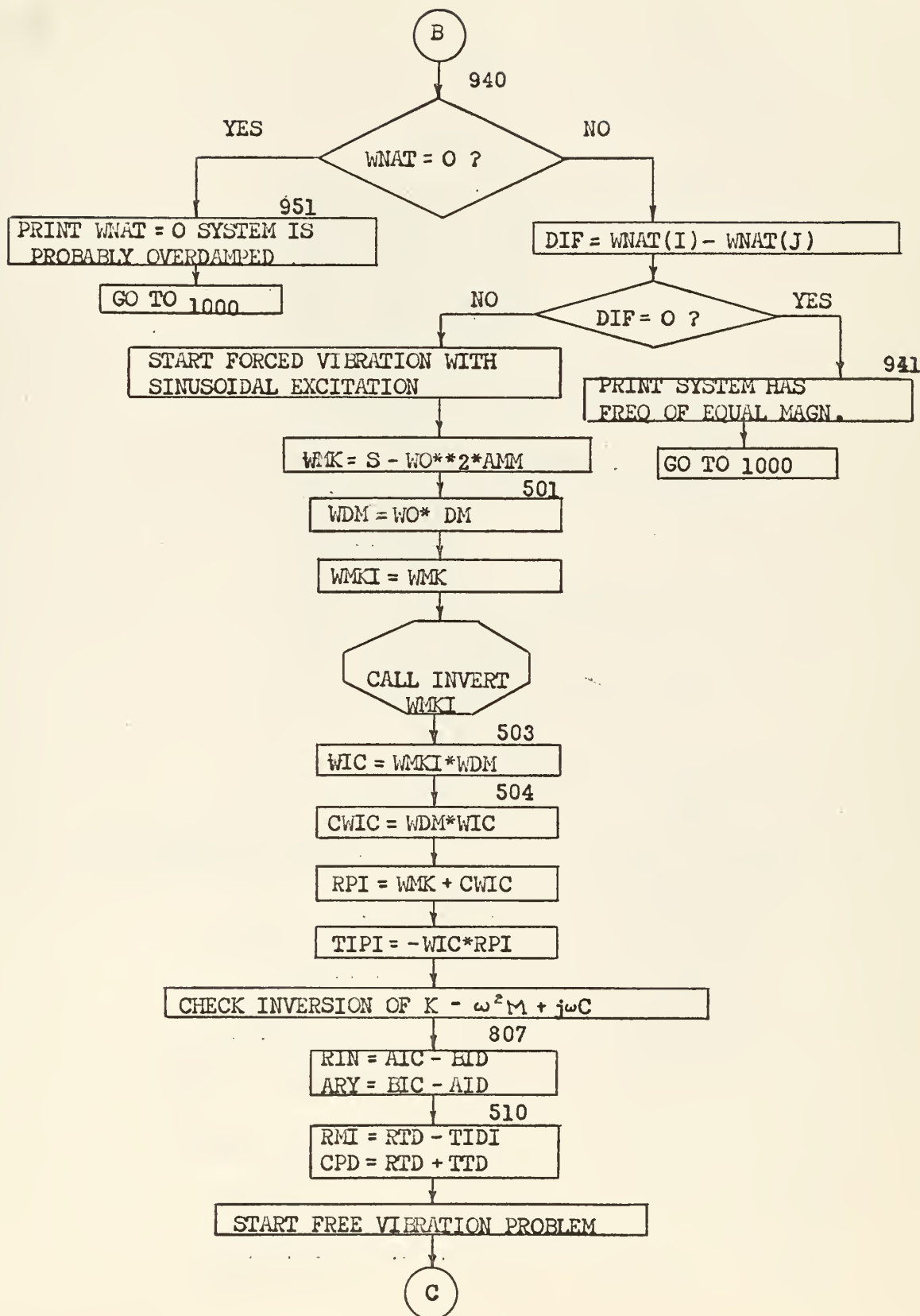
FLOW CHART MAIN BODY PROGRAM VIBSYS



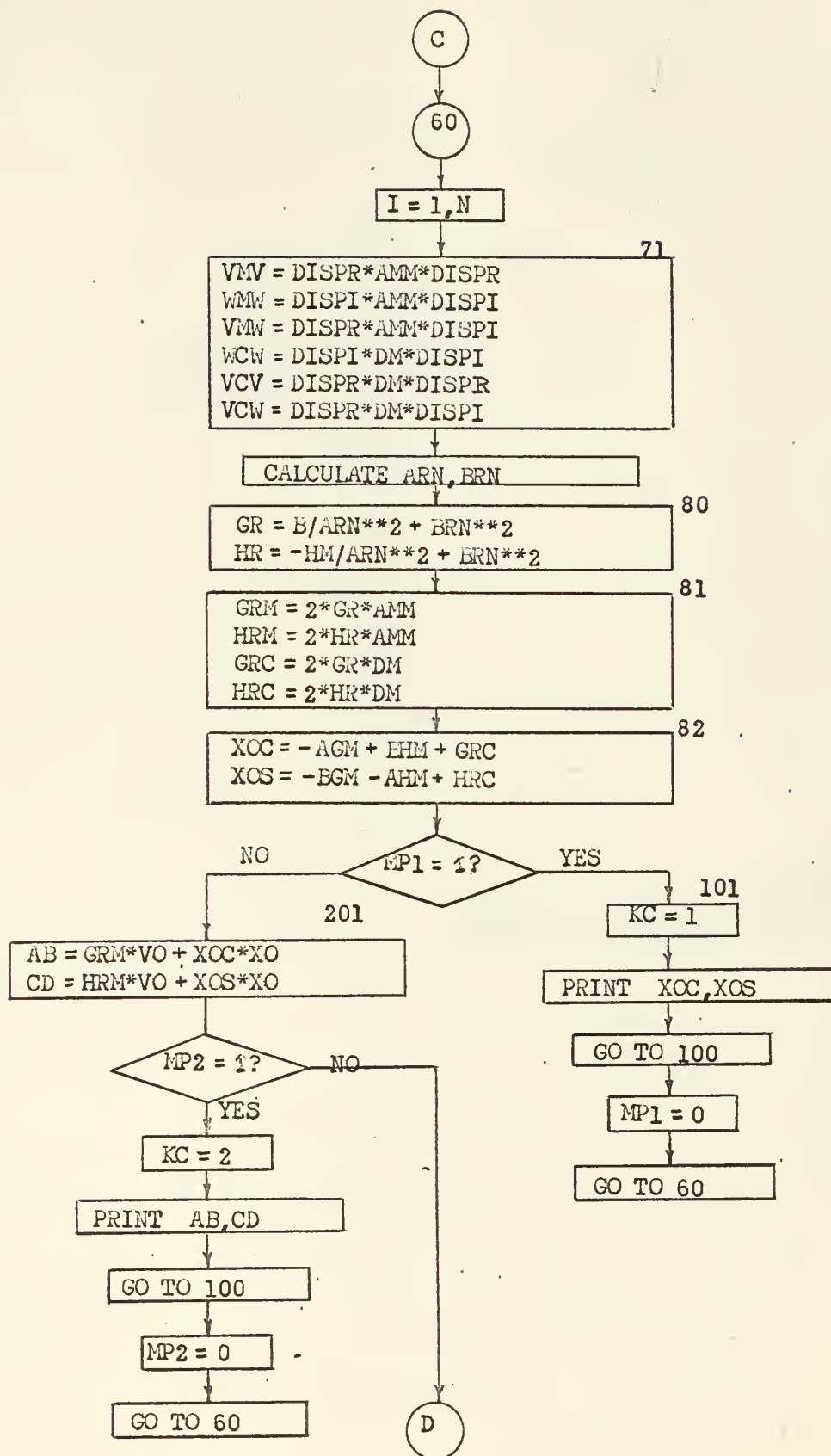






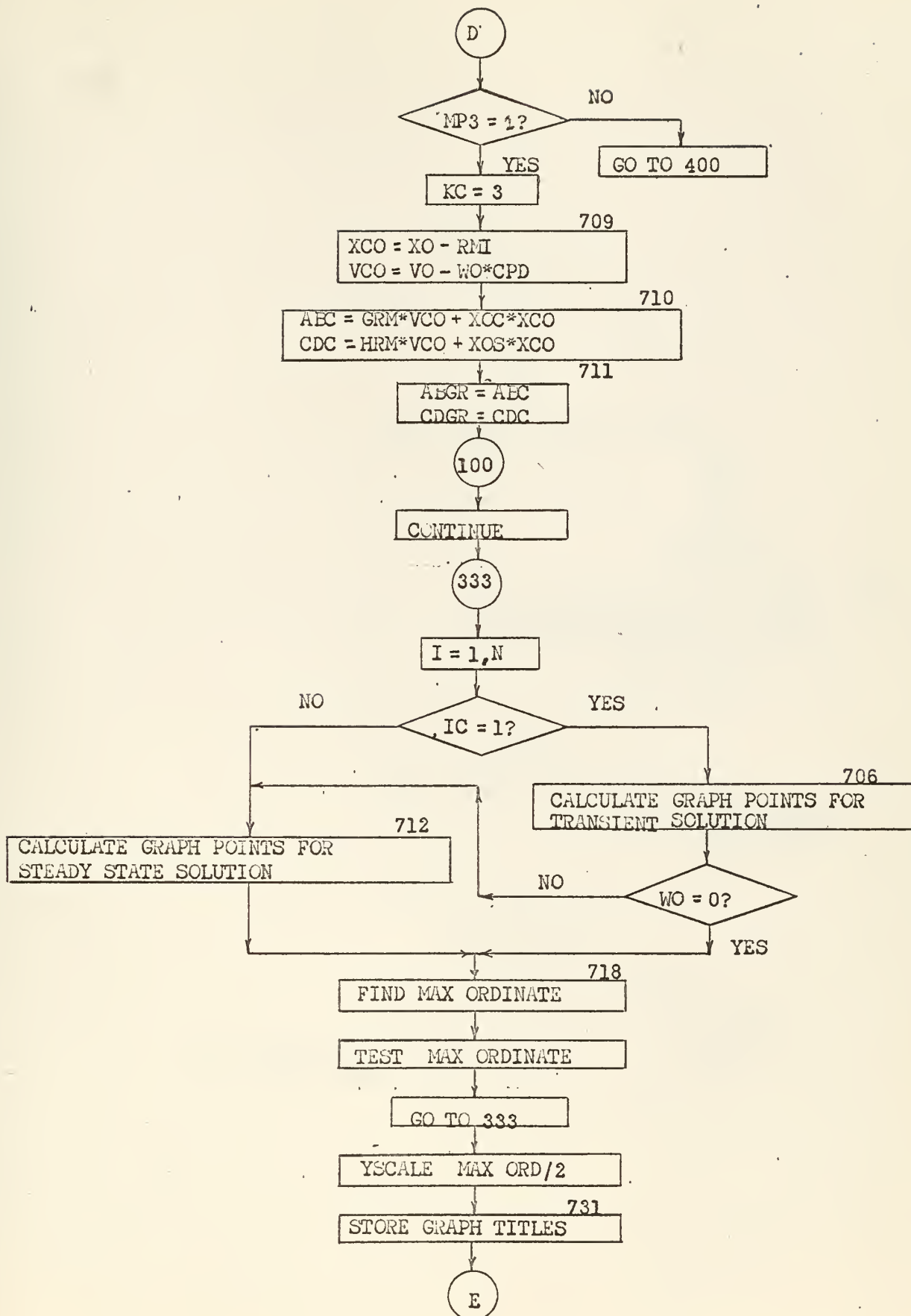




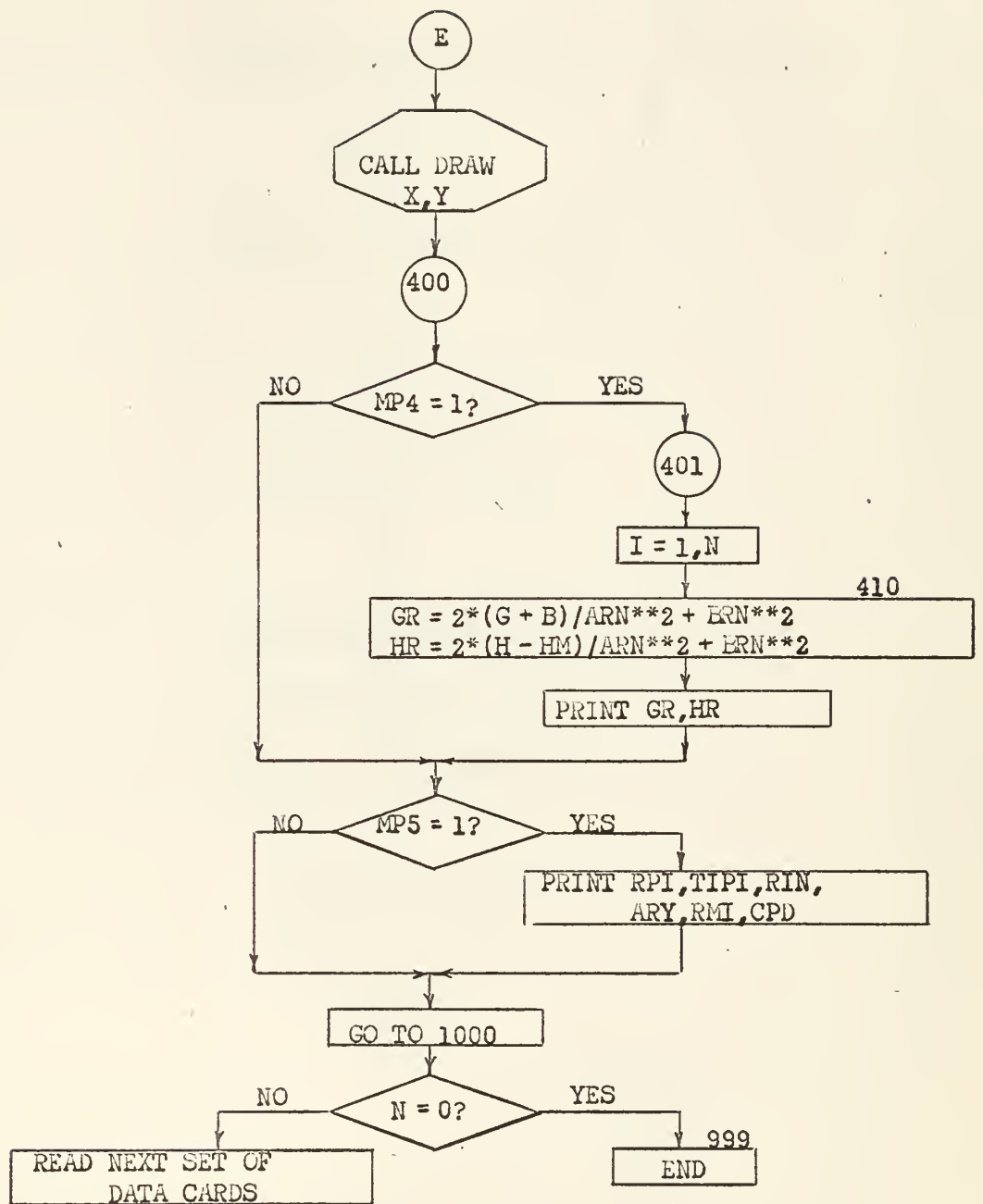






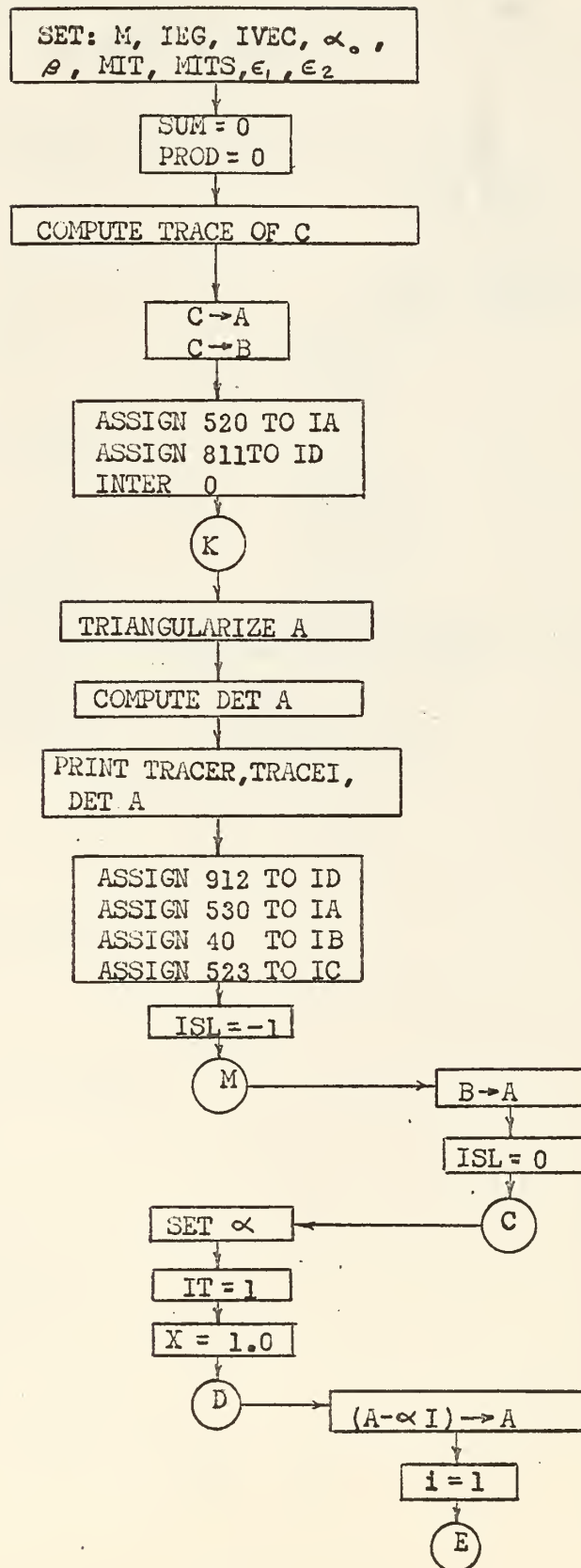




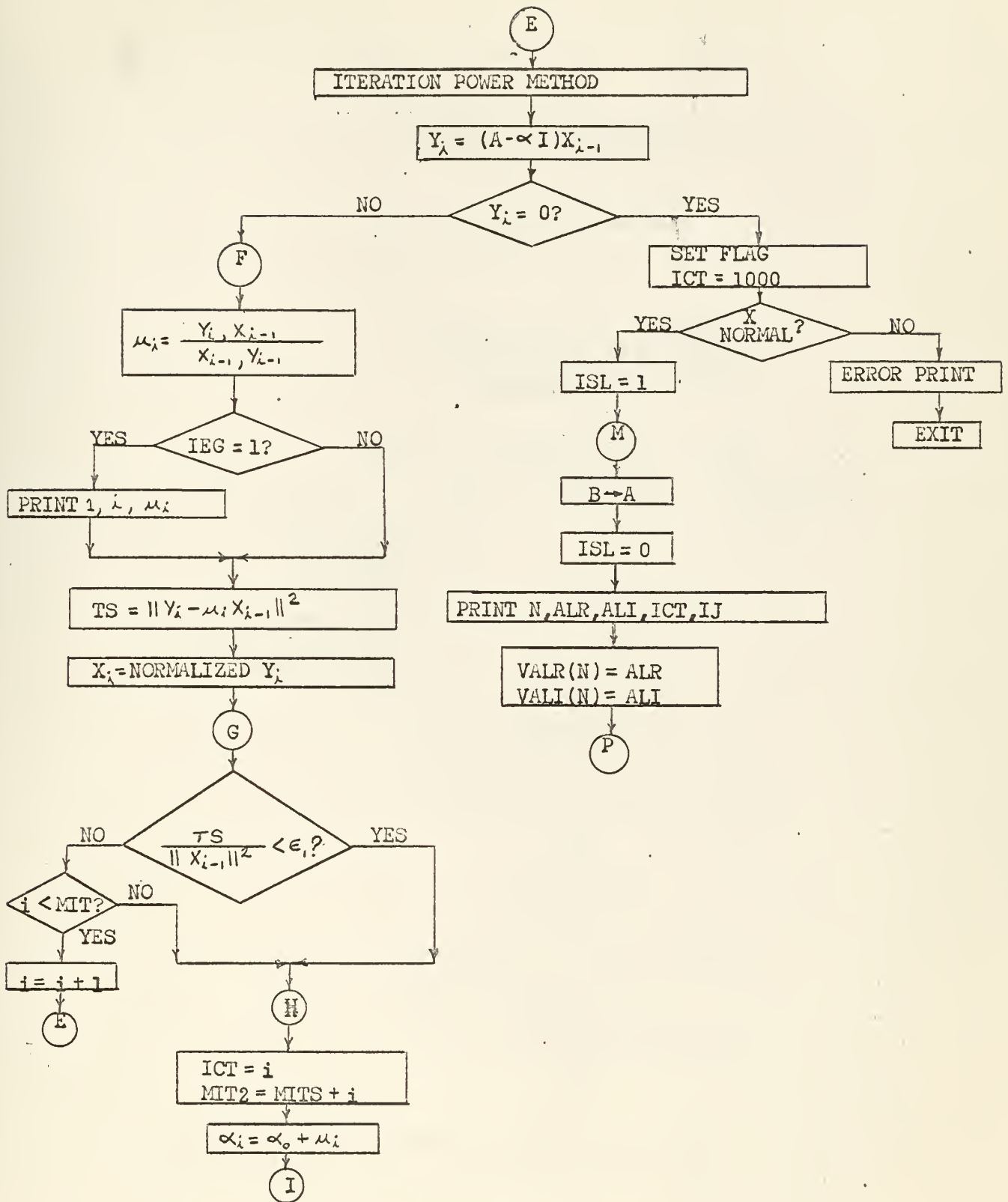




SUBROUTINE MATSUB

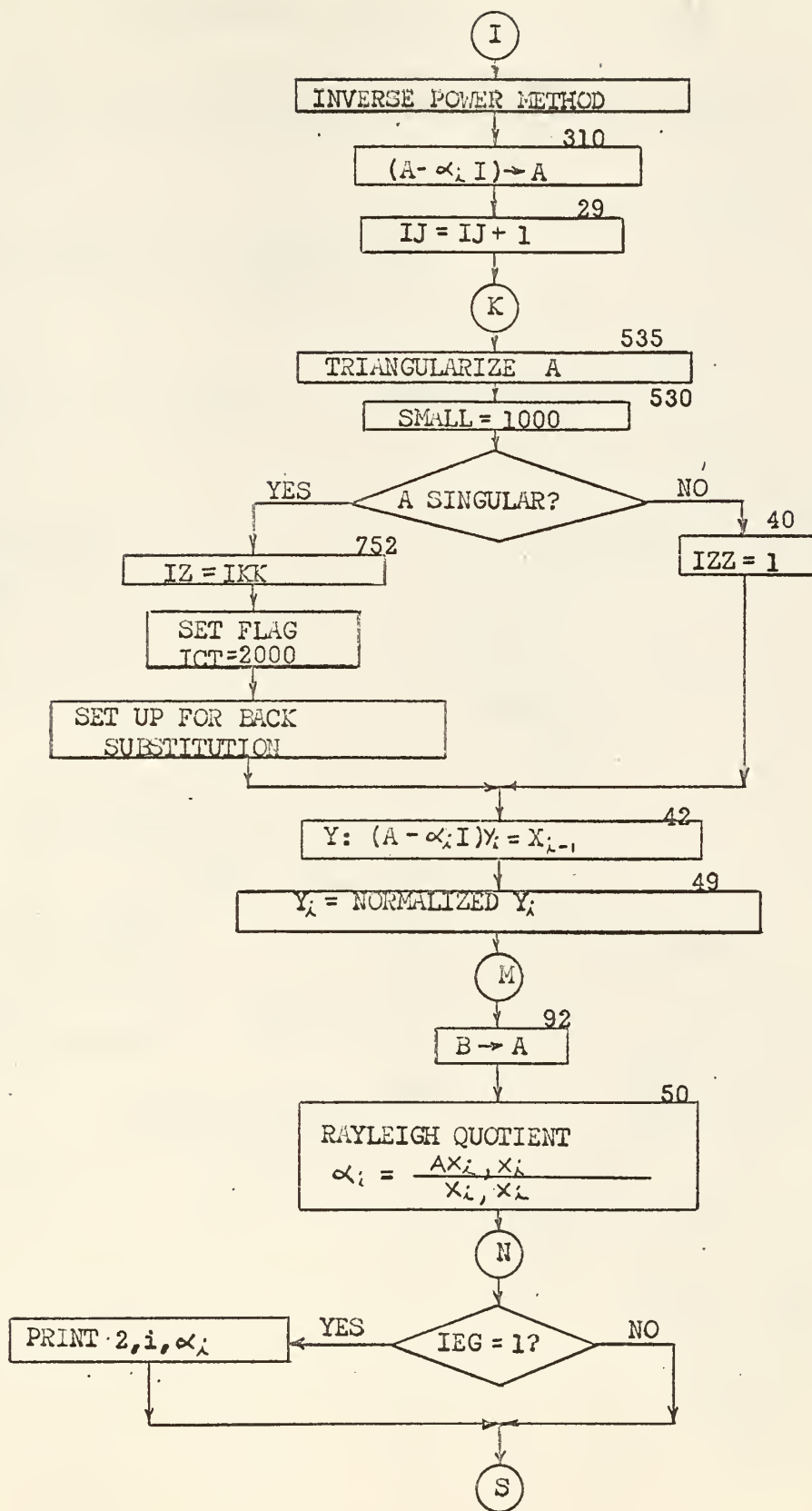




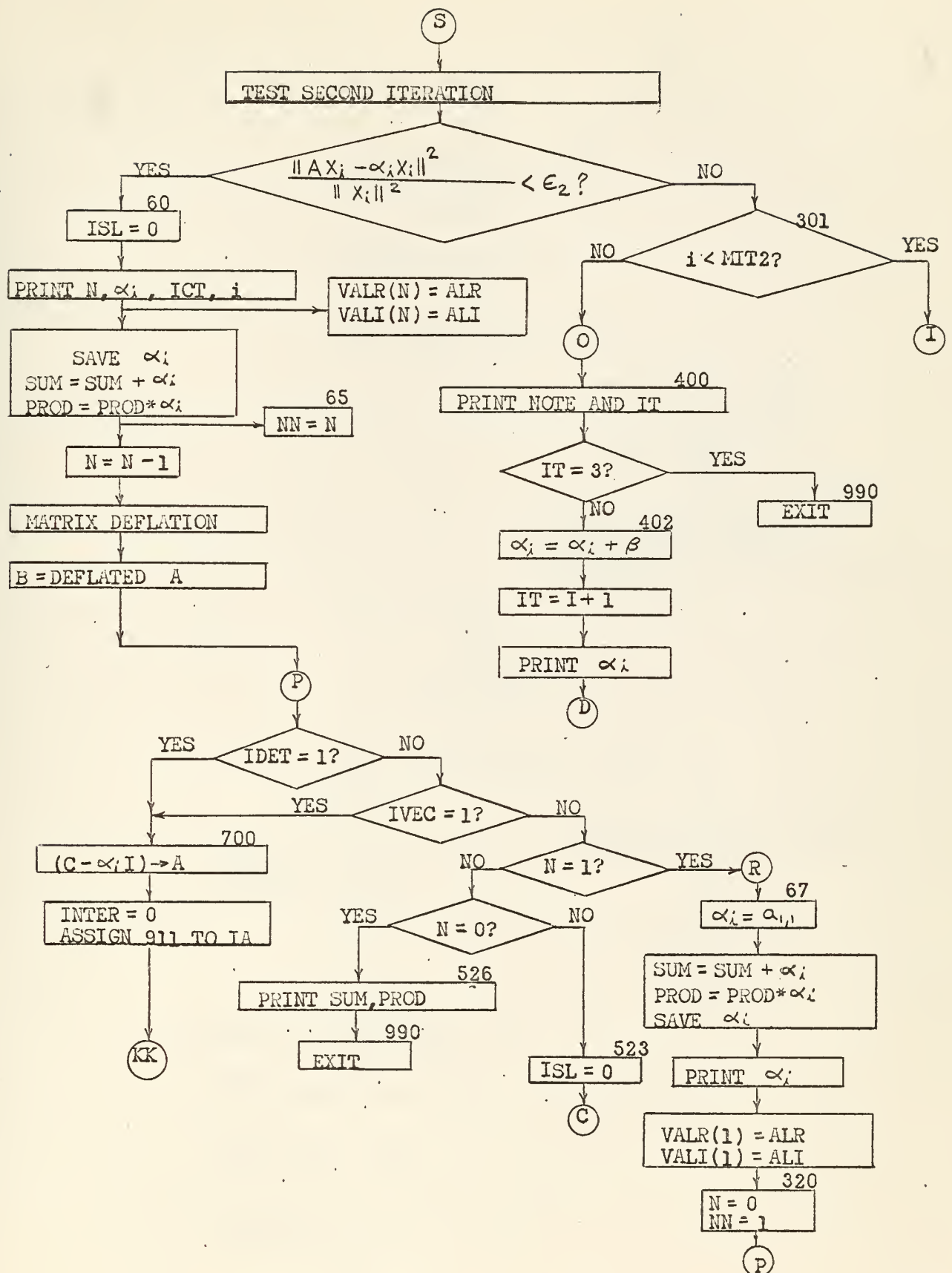




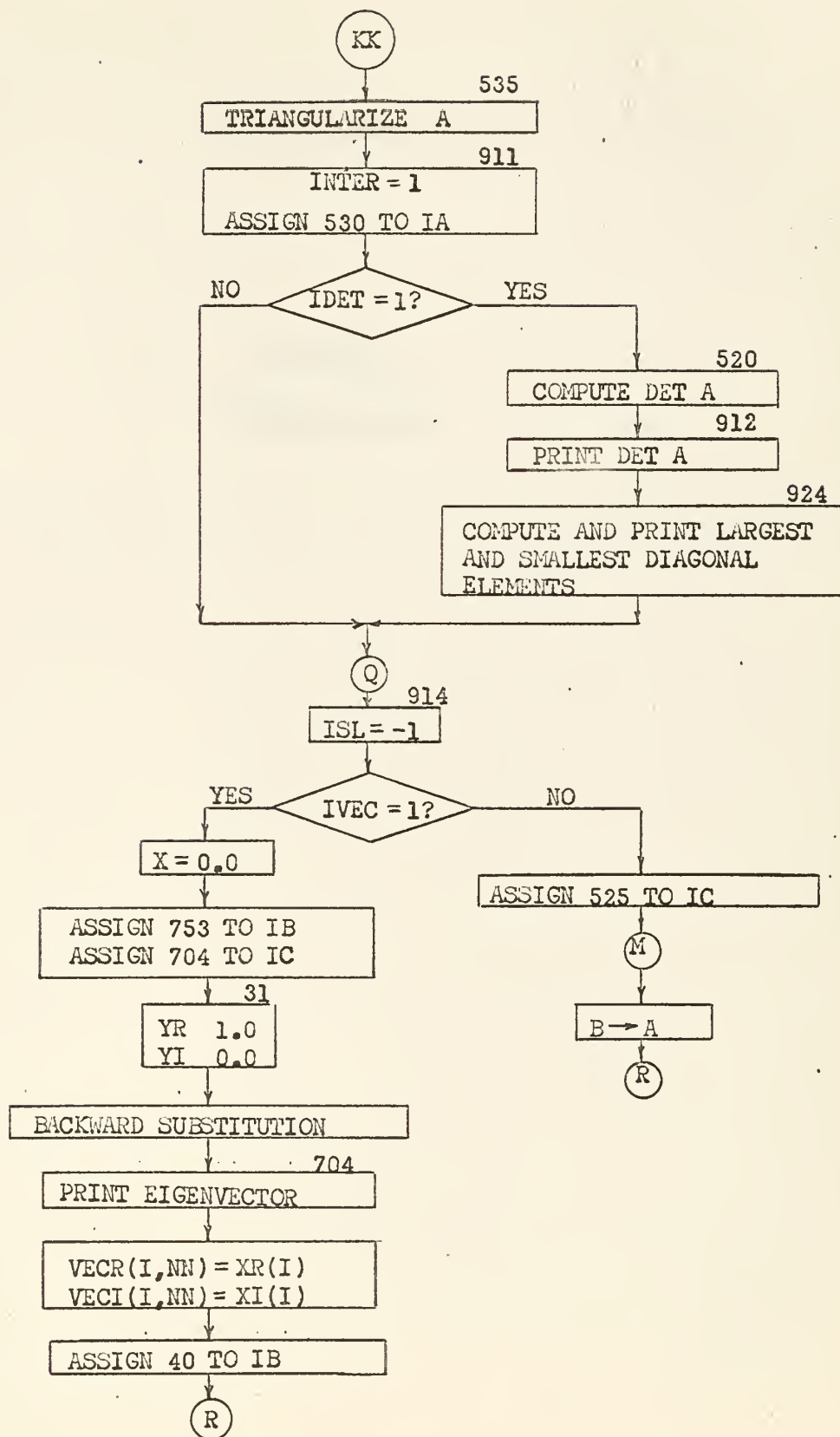














APPENDIX D  
PROGRAM LISTING





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PROGRAM VIBSYS
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DIMENSION AM(10,10),DM(10,10),S(10,10),AMS(10,10),AMD(10,10),VALR
1(20),VALI(20),VECR(20,20),VECI(20,20),AH(20,20),AI(20,20),BR(20,20
2),BI(20,20),UR(20,20),UI(20,20),XR(20),XI(20),YR(20),YI(20),ZR(20)
3,ZI(20)
0COMMON AM,DM,S,AMS,AMD,VALR,VALI,VECR,VECI,AH,AI,BH,BI,UR,UI,XR,XI
1,YR,YI,ZR,ZI
0DIMENSION SPEC(10,10),DISPR(10,10),DISPI(10,10),AMH(10,10),VMV(10)
1,WMW(10),VMW(10),VCV(10),WCV(10),WCM(10),VCW(10),APN(10),BRN(10),G(10,10),
28(10,10),GR(10,10),H(10,10),HM(10,10),HR(10,10),GRM(10,10),HRM(10,
310),GRC(10,10),HRC(10,10),AGM(10,10),RGM(10,10),BHM(10,10),AHM(10,
410),XOC(10,10),XUS(10,10),VAR(10),VAI(10),XOI(10),VO(10),AB(10),CD
5(10),X(900),Y(900),ITITLE(12),WMK(10,10),WMKI(10,10),WDM(10,10),
6WIC(10,10),CWIC(10,10),RPI(10,10),TIP(10,10),DR(10),DI(10),RTD(10
7),ITD(10),ITDI(10),RMI(10),CPD(10),RTDI(10),WNAT(10),RIC(10,10),
8AVAR(10),TEST(10),LAB(10),XCO(10),VCO(10),ABC(10),CDC(10),ABGR(10,
910),CDGR(10,10)
DIMENSION AIC(10,10),RID(10,10),AID(10,10),RIN(10,10),ARY(10,10)
1000 READ 1,N,MP1,MP2,MP3,MP4,MP5,IC,IFC,NPTS
1 FORMAT(9I5)
1 REWIND 6
IF(N)998,998,750
750 PRINT 700
7000FORMAT (16H1 PROGRAM VIBSYS,36X,20HT. J. MIKLOS NHA2,40X,8HMA
11965//,20X,83HMATRIX ANALYSIS OF A MULTI-DEGREE OF FREEDOM VIBRATI
20N SYSTEM WITH VISCOUS DAMPING.//)
PRINT 2,N
2 FORMAT(20H SYSTEM OF ORDER 15)
DO 3 I=1,N
3 READ 4,(AM(I,J),J=1,N)
4 FORMAT(8F10.3)
211 FORMAT(6E20.3)
PRINT 5
5 FORMAT(19H0 INERTIA MATRIX)
DO 6 I=1,N
6 PRINT 211,(AM(I,J),J=1,N)
DO 7 I=1,N
7 READ 4,(UM(I,J),J=1,N)
PRINT 8
8 FORMAT(19H0 DAMPING MATRIX)
DO 9 I=1,N
9 PRINT 211,(DM(I,J),J=1,N)
DO 10 I=1,N
10 READ 4,(SI(I,J),J=1,N)
PRINT 11
11 FORMAT(21H0 STIFFNESS MATRIX)
DO 12 I=1,N
12 PRINT 211,(S(I,J),J=1,N)
210 READ 4,(XO(I),I=1,N)
PRINT 751
751 FORMAT(31H0 THE INITIAL DISPLACEMENTS ARE//)
PRINT 211 (XO(I),I=1,N)
212 READ 4,(VO(I),I=1,N)
PRINT 752
752 FORMAT(40H0 THE INITIAL CONDITIONS OF VELOCITY ARE//)
PRINT 211 (VO(I),I=1,N)
551 READ 4,(UR(I),I=1,N)
HEAD 4,(UI(I),I=1,N)
PRINT 753
753 FORMAT(53H0 THE REAL PART OF THE AMPLITUDE OF THE DRIVING FORCE//)
PRINT 211 (UR(I),I=1,N)

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0063 PRINT 754
0064 754 FORMAT(53H0 THE IMAG PART OF THE AMPLITUDE OF THE DRIVING FORCE//)
0065 PRINT 211(DI(I),I=1,N)
0066 READ 652,WO,STEP
0067 652 FORMAT(2F10.5)
0068 PRINT 755, WO
0069 7550FORMAT(43H0 THE FREQUENCY OF THE FORCING FUNCTION IS F10.4,18HRADI
0070 IANS PER SECOND//)
0071 PRINT 756,STEP
0072 7560FORMAT(44H0 THE STEP SIZE FOR THE GRAPHICAL OPTION IS F5.4,7HSECON
0073 IDS//,10X,17HEND OF INPUT DATA//)
0074 ITERATION PARAMETERS
0075 213 M=2*N
0076 IEG = 0
0077 IVEC = 1
0078 ALRS = 1.0
0079 ALIS = 0.1
0080 GBR = 0.1
0081 GBI = 0.1
0082 IDET = 0
0083 MII = 20
0084 MIIS = 20
0085 EP1 = 1.E-4
0086 EP2 = 1.E-14
0087 LAB(1)=4H M1
0088 LAB(2)=4H M2
0089 LAB(3)=4H M3
0090 LAB(4)=4H M4
0091 LAB(5)=4H M5
0092 LAB(6)=4H M6
0093 LAB(7)=4H M7
0094 LAB(8)=4H M8
0095 LAB(9)=4H M9
0096 LAB(10)=4H M10
0097 END OF INPUT DATA
0098 DO 58 I=1,N
0099 DO 58 J=1,N
0100 58 AM(I,J)=AM(I,J)
0101 13 CALL INVERT (AM,N,D)
0102 DO 14 I=1,N
0103 DO 14 J=1,N
0104 AMS(I,J)=0.0
0105 DO 14 K=1,N
0106 14 AMS(I,J)=AMS(I,J)-AM(I,K)*S(K,J)
0107 DO 15 I=1,N
0108 DO 15 J=1,N
0109 AMD(I,J)=0.0
0110 DO 15 K=1,N
0111 15 AMD(I,J)=AMD(I,J)-AM(I,K)*DM(K,J)
0112 FORM UR MATIIX
0113 DO 161 K=1,M
0114 DO 161 L=1,M
0115 161 UR(K,L)=0.0
0116 DO 16 K=1,N
0117 L=N+K
0118 16 UR(K,L)=1.0
0119 DO 17 I=1,N
0120 DO 17 J=1,N
0121 K=N+I
0122 L=J
0123 17 UR(K,L)=AMS(I,J)
0124 DO 18 I=1,N

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00 18 J=1,N
K=N+1
L=N+J
18 UR(K,L)=AMU(I,J)
00 170 I=1,M
00 170 J=1,M
170 U(I,J)=0.0
19 CALL MATSUB (M,IEG,IVEC,ALNS,ALIS,GBR,GBI,[DET,MIT,MITS,EP1,EP2]
L=0
K=0
00 50 J=1,M
IF(VALI(J))50,50,50
50 K=K+1
VAL(K)=VALR(J)
VAL(K)=VALI(J)
SPEC(K,K)=VALI(J)**2-VALR(J)**2
00 31 JJ=1,N
51 WMAT(JJ)=SURTF(ABSF(SPEC(JJ,JJ)))
00 32 I=1,M
IF(ABSF(VECH(I,J))-1.0)52,55,52
32 CONTINUE
35 L=L+1
35 IF(I-M/2)36,36,38
36 00 40 II=1,N
DISPR(II,L)=VECH(II,L)
40 DISPI(II,L)=VECI(II,L)
GO TO 50
38 00 41 II=1,N
DISPR(II,L)=VECH(II+N,J)
41 DISPI(II,L)=VECI(II+N,J)
50 CONTINUE
PRINT 51
51 FORMAT (30H0 THE NATURAL FREQUENCIES ARE)
52 PRINT 53,(WMAT(J),J=1,N)
53 FORMAT (4E20.8)
PRINT 54
54 FORMAT (28H0 MODAL MATRIX, REAL PART)
00 55 I=1,N
55 PRINT 53,(DISPR(I,J),J=1,N)
PRINT 56
56 FORMAT (28H0 MODAL MATRIX, IMAG PART)
00 57 I=1,N
57 PRINT 53,(DISPI(I,J),J=1,N)
NM1=N-1
00 940 I=1,N
IF(WMAT(I)-0.0001)951,951,940
940 CONTINUE
00 950 I=1,NM1
IP1=I+1
00 950 J=IP1,N
DI=WMAT(I)-WMAT(J)
IF(ABSF(DI)-0.0001)941,941,950
950 CONTINUE
C START FORCED VIBRATION WITH SINUSOIDAL EXCITATION
00 501 I=1,N
00 501 J=1,N
WMK(I,J)=S(I,J)-W0**2*AMM(I,J)
501 WDM(I,J)=W0*DM(I,J)
00 502 I=1,N
00 502 J=1,N
502 WMKI(I,J)=WMK(I,J)
CALL INVERI(WMKI,N,D)

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DO 503 I=1,N
DO 503 J=1,N
WIC(I,J)=0.0
DO 503 K=1,N
503 WIC(I,J)=WIC(I,J)+WMK(I,K)*WDM(K,J)
DO 504 I=1,N
DO 504 J=1,N
CWIC(I,J)=0.0
DO 504 K=1,N
504 CWIC(I,J)=CWIC(I,J)+WDM(I,K)*WIC(K,J)
DO 505 I=1,N
DO 505 J=1,N
505 RPI(I,J)=WMK(I,J)+CWIC(I,J)
CALL INVERT(RPI,N,D)
DO 506 I=1,N
DO 506 J=1,N
TIPI(I,J)=0.0
C CHECK INVERSION OF (K-W**2M+JWC)
DO 506 K=1,N
506 TIPI(I,J)=TIPI(I,J)-WIC(I,K)*RPI(K,J)
DO 806 I=1,N
DO 806 J=1,N
AIC(I,J)=0.0
BID(I,J)=0.0
BIC(I,J)=0.0
AID(I,J)=0.0
DO 806 K=1,N
AIC(I,J)=AIC(I,J)+WMK(I,K)*RPI(K,J)
BID(I,J)=BID(I,J)+WDM(I,K)*TIPI(K,J)
BIC(I,J)=BIC(I,J)+WDM(I,K)*RPI(K,J)
806 AID(I,J)=AID(I,J)+WMK(I,K)*TIPI(K,J)
DO 807 I=1,N
DO 807 J=1,N
RIN(I,J)=AIC(I,J)-BID(I,J)
807 ARY(I,J)=BIC(I,J)+AID(I,J)
DO 510 I=1,N
RTDI(I)=0.0
RTDI(I)=0.0
TTDI(I)=0.0
TTDI(I)=0.0
DO 510 J=1,N
RTDI(I)=RID(I)+RPI(I,J)*DR(J)
RTDI(I)=RTDI(I)+RPI(I,J)*DI(J)
TTDI(I)=TTDI(I)+TIPI(I,J)*DR(J)
TTDI(I)=TTDI(I)+TIPI(I,J)*DI(J)
RMI(I)=RID(I)-TTDI(I)
510 CPD(I)=RID(I)+TTDI(I)
C START OF FREE VIBRATION PROBLEM WITH INITIAL CONDITIONS GIVEN
60 MCI=0
DO 61 I=1,N
61 AVAR(I)=-VAR(I)
DO 100 I=1,N
VMV(I)=0.0
VMW(I)=0.0
VCV(I)=0.0
VCW(I)=0.0
DO 71 J=1,N
DO 71 K=1,N
VMV(I)=VMV(I)+DISPR(K,I)+AMW(K,J)*DISPR(J,I)
VMW(I)=VMW(I)+DISPI(K,I)+AMW(K,J)*DISPI(J,I)

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71  VMW(I)=VMW(I)+DISPR(K,I)*AMM(K,J)*DISPI(J,I)
    VCV(I)=VCV(I)+DISPR(K,I)*DM(K,J)*DISPR(J,I)
    WCV(I)=WCV(I)+DISPI(K,I)*DM(K,J)*DISPI(J,I)
    71  VCV(I)=VCW(I)+DISPR(K,I)*DM(K,J)*DISPI(J,I)
    ARN(I)=0.0
    0ARN(I)=ARN(I)-2.*AVAR(I)*(VMV(I)-WMW(I))-4.0*VAI(I)*VMW(I)+
    1VCV(I)-WCW(I)
    BRN(I)=0.0
    0BRN(I)=BRN(I)+2.0*VAI(I)*(VMV(I)-WMW(I))-4.*AVAR(I)*VMW(I)+
    12.0*VCW(I)
    DO 80 J=1,N
    DO 80 K=1,N
    GR(J,K)=0.0
    HR(J,K)=0.0
    G(J,K)=0.0
    H(J,K)=0.0
    B(J,K)=0.0
    HM(J,K)=0.0
    G(J,K)=G(J,K)+DISPR(J,I)*DISPR(K,I)-DISPI(J,I)*DISPI(K,I)
    H(J,K)=BRN(I)*G(J,K)
    G(J,K)=ARN(I)*G(J,K)
    B(J,K)=B(J,K)+DISPI(J,I)*DISPR(K,I)+DISPR(J,I)*DISPI(K,I)
    HM(J,K)=ARN(I)*B(J,K)
    B(J,K)=BRN(I)*B(J,K)
    GR(J,K)=(G(J,K)+B(J,K))/(ARN(I)**2+BRN(I)**2)
    80  HR(J,K)=(H(J,K)-HM(J,K))/(ARN(I)**2+BRN(I)**2)
    DO 81 J=1,N
    DO 81 K=1,N
    GRM(J,K)=0.0
    HRM(J,K)=0.0
    GRC(J,K)=0.0
    HRC(J,K)=0.0
    DO 81 L=1,N
    GRM(J,K)=GRM(J,K)+2.0*GR(J,L)*AMM(L,K)
    HRM(J,K)=HRM(J,K)+2.0*HR(J,L)*AMM(L,K)
    GRC(J,K)=GRC(J,K)+2.0*GR(J,L)*DM(L,K)
    HRC(J,K)=HRC(J,K)+2.0*HR(J,L)*DM(L,K)
    81  DO 82 J=1,N
    DO 82 K=1,N
    XOC(J,K)=0.0
    AGM(J,K)=AVAR(I)*GRM(J,K)
    BHM(J,K)=VAI(I)*HRM(J,K)
    BGM(J,K)=VAI(I)*GRC(J,K)
    AHM(J,K)=AVAR(I)*HRM(J,K)
    XDC(J,K)=-AGM(J,K)+BHM(J,K)+GRC(J,K)
    82  XOS(J,K)=-BGM(J,K)-AHM(J,K)+HRC(J,K)
    IF(MPI)999,201,101
    101 KC=1
    PRINT 83,VAI(I)
    83  FORMAT(39H0 THE COEFFICIENT MATRIX OF VEL X COS(F8.3,2HT))
    DO 84 J=1,N
    84  PRINT 85,(GRM(J,K),K=1,N)
    85  FORMAT(4E20.8)
    PRINT 86,VAI(I)
    86  FORMAT(39H0 THE COEFFICIENT MATRIX OF X COS(F8.3,2HT))
    DO 87 J=1,N
    87  PRINT 85,(XOC(J,K),K=1,N)
    PRINT 88,VAI(I)
    88  FORMAT(39H0 THE COEFFICIENT MATRIX OF VEL X SIN(F8.3,2HT))
    DO 89 J=1,N
    89  PRINT 85,(HRM(J,K),K=1,N)

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0311 PRINT 90,VAI(I)
0312 90 FORMAT(35H0 THE COEFFICIENT MATRIX OF X SIN(F8.3,2HT))
0313 DO 91 J=1,N
0314 91 PRINT 85,(XOS(J,K),K=1,N)
0315 GO TO 100
0316 201 DO 202 J=1,N
0317 AB(J)=0.0
0318 CD(J)=0.0
0319 DO 202 K=1,N
0320 AB(J)=AB(J)+GRM(J,K)*VO(K)+XOC(J,K)*XO(K)
0321 202 CD(J)=CD(J)+HRM(J,K)*VO(K)+XOS(J,K)*XO(K)
0322 IF(MP2)999,301,203
0323 203 KC=2
0324 PRINT 204 VAR(I),VAI(I)
0325 204 FORMAT(35H0 THE COEFFICIENT COLUMN FOR EXP(F8.3,6HT)COS(F8.3,2HT))
0326 205 PRINT 206(AH(J),J=1,N)
0327 206 FORMAT(6E20.4)
0328 PRINT 207 VAR(I),VAI(I)
0329 207 FORMAT(35H0 THE COEFFICIENT COLUMN FOR EXP(F8.3,6HT)SIN(F8.3,2HT))
0330 208 PRINT 208(CD(J),J=1,N)
0331 GO TO 100
0332 301 IF(MP3)999,400,302
0333 302 KC=3
0334 DO 709 J=1,N
0335 XCO(J)=0.0
0336 VCO(J)=0.0
0337 XCO(J)=XO(J)-RMI(J)
0338 709 VCO(J)=VO(J)+WO*CPD(J)
0339 DO 710 J=1,N
0340 ABC(J)=0.0
0341 CDC(J)=0.0
0342 DO 710 K=1,N
0343 ABC(J)=ABC(J)+GRM(J,K)*VCO(K)+XOC(J,K)*XCO(K)
0344 710 CDC(J)=CDC(J)+HRM(J,K)*VCO(K)+XOS(J,K)*XCO(K)
0345 DO 711 J=1,N
0346 ABGR(J,I)=ABC(J)
0347 711 CDGR(J,I)=CDC(J)
0348 100 CONTINUE
0349 IF(KC-2)150,160,333
0350 150 MP1=0
0351 PRINT 102
0352 102 FORMAT(47H0 FREE VIBRATION OPTION NO. 1 HAS BEEN EXECUTED//)
0353 GO TO 60
0354 160 MP2=0
0355 PRINT 103
0356 103 FORMAT(47H0 FREE VIBRATION OPTION NO. 2 HAS BEEN EXECUTED//)
0357 GO TO 60
0358 333 DO 360 I=1,N
0359 IF(IC)999,717,712
0360 712 SI=0.0
0361 DO 713 K=1,NPTS
0362 Y(K)=0.0
0363 X(K)=ST
0364 DO 706 J=1,N
0365 OY(K)=Y(K)+ABGR(I,J)*EXP(VAR(J)*X(K))+COSF(VAI(J)*X(K))+CDGR(I,J)*
0366 1EXP(VAR(J)*X(K))*SINF(VAI(J)*X(K))
0367 706 CONTINUE
0368 713 ST=SI+STEP
0369 IF(WO)999,718,717
0370 717 ST=0.0
0371 DO 716 K=1,NPTS
0372 X(K)=ST

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Y(K)=Y(K)+MM1(I)*COSF(W0*X(K))-CP0(I)*SINF(W0*X(K))
STEP=STEP+1
716 CONTINUE
718 TEST(I)=0.0
DO 715 J=1,NPTS
IF(ABSF(Y(J))-TEST(I))715,715,714
714 TEST(I)=ABSF(Y(J))
715 CONTINUE
WRITE TAPE 6,Y
360 CONTINUE
SCALE=0.0
DO 721 I=1,N
IF(TEST(I)-SCALE)721,721,722
722 SCALE=TEST(I)
721 CONTINUE
YSCALE=SCALE/2.0
REWIND 6
DO 730 I=1,N
MODCURV=0
LABEL=LAD(I)
DO 731 K=1,12
731 ITITLE(K)=8H
IF(W0)733,732,733
732 ITITLE(1)=8H MIKLOS,
ITITLE(2)=8H T.J.
ITITLE(3)=8H JOH
ITITLE(4)=8H BOX M
ITITLE(5)=8H 0199
ITITLE(6)=8H HUX M
ITITLE(7)=8H FREE VI
ITITLE(8)=8H HRRATION
ITITLE(9)=8H HRRATION
GO TO 734
733 IF(IC)999,735,736
736 ITITLE(1)=8H MIKLOS,
ITITLE(2)=8H T.J.
ITITLE(3)=8H JOH
ITITLE(4)=8H BOX M
ITITLE(7)=8H TRANSIE
ITITLE(8)=8HNT PLUS
ITITLE(9)=8HSTRAUY S
ITITLE(10)=8H TATE
GO TO 734
735 ITITLE(1)=8H MIKLOS,
ITITLE(2)=8H T.J.
ITITLE(3)=8H JOH
ITITLE(4)=8H HUX M
ITITLE(7)=8H STEADY
ITITLE(8)=8HSTATE VI
ITITLE(9)=8H HRRATION
734 READ TAPE 6,Y
OCALL DRAW(NPTS,X,Y,MODCURV,0,LABEL,ITITLE,0,YSCALE,2.0,2.0,8,4,1,
1,LA5T)
730 CONTINUE
400 IF(MP4)999,500,401
401 DO 450 I=1,N
DO 410 J=1,N
DO 410 K=1,N
GH(J,K)=0.0
HH(J,K)=0.0
G(J,K)=0.0
H(J,K)=0.0
B(J,K)=0.0

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HM(J,K)=0.0
G(J,K)=G(J,K)+DISPR(J,I)*DISPR(K,I)-DISPI(J,I)*DISPI(K,I)
H(J,K)=HRN(I)*G(J,K)
G(J,K)=ARV(I)*G(J,K)
B(J,K)=B(J,K)+DISPI(J,I)*DISPR(K,I)+DISPR(J,I)*DISPI(K,I)
HM(J,K)=HRN(I)*B(J,K)
B(J,K)=BRV(I)*B(J,K)
GR(J,K)=2.0*(G(J,K)+B(J,K))/(ARV(I)*2+BRN(I)*2)
HR(J,K)=2.0*(H(J,K)-HM(J,K))/(ARV(I)*2+BRN(I)*2)
410 PRINT 411 I
4110FORMAT(77H0 THE COEFFICIENT MATRIX OF THE CONVOLUTION INTEGRAL OF
      THE COS TERM FOR MODELS,1X,2HIS)
      DO 412 J=1,N
412 PRINT 413 (GR(J,K),K=1,N)
413 FORMAT (6E20.8)
      PRINT 414 I
4140FORMAT(77H0 THE COEFFICIENT MATRIX OF THE CONVOLUTION INTEGRAL OF
      THE SIN TERM FOR MODELS,1X,2HIS)
      DO 415 J=1,N
415 PRINT 415(HR(J,K),K=1,N)
450 CONTINUE
      PRINT 104
104 FORMAT(49H0 FORCED VIBRATION OPTION NO. 4 HAS BEEN EXECUTED//)
500 IF(IMP5)999,999,800
800 PRINT 801
801 FORMAT(30H0 THE REAL PART OF THE INVERSE)
      DO 802 I=1,N
802 PRINT 803 (MPI(I,J),J=1,N)
803 FORMAT(8E15.4)
      PRINT 804
804 FORMAT(30H0 THE IMAG PART OF THE INVERSE)
      DO 805 I=1,N
805 PRINT 805 (IPI(I,J),J=1,N)
      PRINT 808
808 FORMAT(38H0 THE REAL PART OF THE IDENTITY MATRIX)
      DO 809 I=1,N
809 PRINT 810 (KIN(I,J),J=1,N)
810 FORMAT(8E15.4)
      PRINT 811
811 FORMAT(38H0 THE IMAG PART OF THE IDENTITY MATRIX)
      DO 812 I=1,N
812 PRINT 810 (ARY(I,J),J=1,N)
511 PRINT 512 W0
512 FORMAT(32H0 THE COEFFICIENT COLUMN OF COS(P6.3,2HT))
      PRINT 513(HMI(I),I=1,N)
513 FORMAT(6E20.4)
      PRINT 514 W0
514 FORMAT(32H0 THE COEFFICIENT COLUMN OF SIN(P6.3,2HT))
      PRINT 513(CPD(I),I=1,N)
105 PRINT 106
106 FORMAT(49H0 FORCED VIBRATION OPTION NO. 5 HAS BEEN EXECUTED//)
      GO TO 999
941 PRINT 942
9420FORMAT(90H0 SYSTEM CONTAINS TWO NATURAL FREQUENCIES OF EQUAL MAGNI
      TITUDE. PROGRAM HAS BEEN TERMINATED.)
      GO TO 1000
951 PRINT 952
9520FORMAT(100H0 PROGRAM HAS BEEN TERMINATED DUE TO A NATURAL FREQUENC
      Y OF ZERO MAGNITUDE. SYSTEM IS PROBABLY OVERDAMPED.)
999 GO TO 1000
998 END
      SUBROUTINE INVERT (A,N,D)

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C      PROGRAM FOR FINDING THE INVERSE OF A NXN MATRIX
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C      DIMENSION AM(10,10),DM(10,10),S(10,10),AMS(10,10),AMD(10,10),VALR
1(20),VALI(20),VECR(20,20),VECI(20,20),AR(20,20),AI(20,20),BR(20,20)
2),BI(20,20),UR(20,20),UI(20,20),XR(20),XI(20),YR(20),ZI(20)
3,ZI(20)
C      COMMON AM,DM,S,AMS,AMD,VALR,VALI,VECR,VECI,AR,AI,BR,BI,UR,UI,XR,XI
1,YR,YI,ZR,ZI
C      DIMENSION A(10,10),L(10),M(10)
C      SEARCH FOR LARGEST ELEMENT
D=1.0
D080 K=1,N
L(K)=K
M(K)=K
BIGA=A(K,K)
D020 I=K,N
D020 J=K,N
IF(ABS(BIGA)-ABS(A(I,J))) 10,20,20
10 BIGA=A(I,J)
L(K)=I
M(K)=J
20 CONTINUE
C      INTERCHANGE ROWS
J=L(K)
IF(L(K)-K) 35,35,25
25 D030 I=1,N
HOLD=-A(K,I)
A(K,I)=A(J,I)
A(J,I)=HOLD
30 A(J,I)=HOLD
C      INTERCHANGE COLUMNS
35 I=M(K)
IF(M(K)-K) 45,45,37
37 D040 J=1,N
HOLD=-A(J,K)
A(J,K)=A(I,K)
A(I,K)=HOLD
40 A(J,I)=HOLD
C      DIVIDE COLUMN BY MINUS PIVOT
45 D055 I=1,N
46 IF(I-K)50,55,50
50 A(I,K)=A(I,K)/(-A(K,K))
55 CONTINUE
C      REDUCE MATRIX
D065 I=1,N
D065 J=1,N
56 IF(I-K) 57,65,57
57 IF(J-K) 60,65,60
60 A(I,J)=A(I,K)*A(K,J)+A(I,J)
65 CONTINUE
C      DIVIDE ROW BY PIVOT
D075 J=1,N
68 IF(J-K)70,75,70
70 A(K,J)=A(K,J)/A(K,K)
75 CONTINUE
C      CONTINUED PRODUCT OF PIVOTS
D=D*A(K,K)
C      REPLACE PIVOT BY RECIPROCAL
A(K,K)=1.0/A(K,K)
80 CONTINUE
C      FINAL ROW AND COLUMN INTERCHANGE
K=N
100 K=(K-1)
IF(K) 150,150,103
103 I=L(K)

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105 IF(I-K) 120,120,105
106 DO 110 J=1,N
107 HOLD=A(J,K)
108 A(J,K)=-A(J,I)
109 A(J,I)=HOLD
110 J=M(K)
111 IF(J-K) 100,100,125
112 DO 130 I=1,N
113 HOLD=A(K,I)
114 A(K,I)=-A(J,I)
115 A(J,I)=HOLD
116 GO TO 100
117 RETURN
118 END
0SUBROUTINE MATSUB (M,IEG,IVEC,ALRS,ALIS,GHR,GHI,IDET,MIT,MITS,EPI,
119 IEP2)
0DIMENSION AM(10,10),DM(10,10),S(10,10),AMS(10,10),AMD(10,10),VALR
120 1(20),VALI(20),VECR(20,20),VECI(20,20),AR(20,20),AI(20,20),BR(20,20
20 2),BI(20,20),CR(20,20),CI(20,20),XR(20,20),XI(20,20),ZR(20
30 3),ZI(20)
0COMMON AM,DM,S,AMS,AMD,VALR,VALI,VECR,VECI,AR,AI,BR,BI,CR,CI,XR,XI
121 1,YR,YI,ZR,ZI
122 I IF=1
123 I WO=2
124 N=1
125 SUMR=0.0
126 SUMI=0.0
127 PRDR=1.0
128 PRDI=0.0
129 TRACER=0.0
130 TRACEI=0.0
131 DO 450 I=1,N
132 TRACER=TRACER+CR(I,I)
133 TRACEI=TRACEI+CI(I,I)
450 SEI UP MATRICES
134 DO 519 I=1,N
135 DO 519 J=1,N
136 BR(I,J)=CR(I,J)
137 AR(I,J)=CH(I,J)
138 BI(I,J)=CI(I,J)
519 AI(I,J)=CI(I,J)
139 EVALUATE DETERMINANT
140 ASSIGN 520 TO IA
141 ASSIGN 811 TO ID
142 MM=M
143 INIER=0
144 GO TO 535
520 DETR=1.0
145 DO 522 K=1,M
146 DI=0.0
147 I1=DETR*AR(K,K)-DETR*AI(K,K)
148 DETI=DETR*AI(K,K)+DETR*AR(K,K)
522 DETR=I1
149 INIER=XMODF(INIER,2)
150 IF (INIER) 1000,917,810
810 DETR=-DETR
151 DETI=-DETI
917 GO TO ID
811 PRINT 557,TRACER,TRACEI,DETR,DETI
557 FORMAT (19H TRACE OF MATRIX= 2E18.8,
125H DETERMINANT OF MATRIX= 2E18.8)
152 ASSIGN 912 TO ID

```



```

        ASSIGN 530 TO IA
        ASSIGN 40 TO IB
        ASSIGN 523 TO IC
        ISL=-1
        GO TO 92
023 ISL=0
C      EIGENVALUE GUESS OR ORIGIN TRANSLATION
    9  ALR=ALRS
      ALI=ALIS
      II=1
C      EIGENVECTOR GUESS
    403 DO 504 I=1,N
      XR(I)=1.0
    504 XI(I)=0.0
      4 DO 5 I=1,N
        AR(I,I)=AR(I,I)-ALR
        AI(I,I)=AI(I,I)-ALI
    5  FIRST ITERATION - POWER METHOD
      IJ=1
    10 BIG=0.
C      COMPUTE Y=(A-ALPHA)*X
      DO 13 I=1,N
        YR(I)=0.
        YI(I)=0.
        DO 11 J=1,N
          YR(I)=YR(I)+AR(I,J)*XR(J)-AI(I,J)*XI(J)
          YI(I)=YI(I)+AI(I,J)*XR(J)+AR(I,J)*XI(J)
          AM=YR(I)**2+YI(I)**2
          IF (AM-BIG) 13,13,12
        11 J=1,N
      13 I=1,N
    12 BIG=AM
      JJ=1
    13 CONTINUE
      IF (BIG) 109,106,109
C      EXACT EIGENVALUE AND EIGENVECTOR - Y=0. FLAG=1000
    106 ICI=1000
      DO 108 I=1,N
        JJ=1
        IF (XR(I)-1.0) 108,118,108
      108 I=1
      GO TO 92
    108 CONTINUE
      PRINT 650
    650 FORMAT (4H ERROR, EIGENVECTOR NOT NORMALIZED IN METHOD 1.)

      GO TO 990
C      MU RAYLEIGH QUOTIENT - (Y,X)/(X,X)=MU
    109 RQNR=0.
      RQNI=0.
      RQU=0.
      DO 14 I=1,N
        RQNR=RQNR+XR(I)*YR(I)+XI(I)*YI(I)
        RQNI=RQNI+XR(I)*YI(I)+XI(I)*YR(I)
      14 RQD=RQD+XR(I)**2+XI(I)**2
      AMUR=RQNR/RQD
      AMUI=RQNI/RQD
      AMM=AMUR**2+AMUI**2
      IF (ICG) 1000,81,80
    80 ALRC=AMUR+ALR
      ALIC=AMUI+ALI

```



```

0681 PRINT 300, IONE, IJ, ALRC, ALIC
0682
0683 300 FORMAT (2I4, 2E20.8)
0684 TEST FIRST ITERATION
0685 C MAGNITUDE OF (Y-MU*X)=TS
0686 81 TS=0.
0687 DO 15 I=1,N
0688 150 TS=TS+(YR(I)-AMUR*XR(I)+AMUI*XI(I))*2+
0689 1(YI(I)-AMUI*XI(I)-AMUI*XR(I))*2
0690 C NORMALIZATION
0691 DO 16 I=1,N
0692 16 XR(I)=(YR(IJ)+YR(I)-YI(IJ)+YI(I))/BIG
0693 16 XI(I)=(YR(IJ)+YI(I)-YI(IJ)+YR(I))/BIG
0694 XR(IJ)=1.0
0695 XI(IJ)=0.0
0696 111 IF (IS/RND-EPI) 20,20,18
0697 18 IF (IJ-MIT) 19,20,20
0698 19 IJ=IJ+1
0699 GO TO 10
0700 C SECOND ITERATION - INVERSE POWER METHOD
0701 20 ICI=IJ
0702 MI12=MIT+IJ
0703 ALH=AMUR*ALH
0704 ALI=AMUI*ALI
0705 MM=EN
0706 DO 310 I=1,N
0707 AR(I,I)=AR(I,I)-AMUR
0708 310 AI(I,I)=AI(I,I)-AMUI
0709 GO TO 29
0710 99 DO 100 I=1,N
0711 AR(I,I)=AR(I,I)-ALH
0712 100 AI(I,I)=AI(I,I)-ALI
0713 29 IJ=IJ+1
0714 C GAUSSIAN ELIMINATION - (A-ALPHA)*Y=X
0715 535 DO 27 I=2,MM
0716 IM1=I-1
0717 DO 27 J=1,IM1
0718 21 FM=AR(I,J)*2+AI(I,J)*2
0719 SM=AR(J,J)*2+AI(J,J)*2
0720 IF (FM-SM) 24,24,22
0721 C ROW INTERCHANGE - IF NECESSARY
0722 22 DO 23 K=J,MM
0723 I1=AR(J,K)
0724 I2=AI(J,K)
0725 AR(J,K)=AR(I,K)
0726 AI(J,K)=AI(I,K)
0727 AR(I,K)=I1
0728 AI(I,K)=I2
0729 I1=XR(J)
0730 I2=XI(J)
0731 XR(J)=XR(I)
0732 XI(J)=XI(I)
0733 XR(I)=I1
0734 XI(I)=I2
0735 I1=FM
0736 FM=SM
0737 SM=I1
0738 INTER=INTER+1
0739 24 IF (SM) 25,27,25
0740 25 IF (FM) 90,27,90
0741 C TRIANGULARIZATION
0742 90 TR=(AR(I,J)+AR(J,J)+AI(I,J)+AI(J,J))/SM
    TI=(AR(J,J)+AI(I,J)-AR(I,J)+AI(J,J))/SM

```





```

DO 26 K=J,MM
  AR(I,K)=AR(I,K)-RR*AR(J,K)+RI*AI(J,K)
26  AI(I,K)=AI(I,K)-RR*AI(J,K)+RI*AR(J,K)
  AR(I,J)=0.
  AI(I,J)=0.
  XR(I)=XR(I)-RR*XR(J)+RI*XI(J)
  XI(I)=XI(I)-RR*XI(J)+RI*XR(J)
27  CONTINUE
  GO TO 1A
530  SMALL=1000.
  DO 28 K=1,MM
    IKK=K
    I1=AR(K,K)*2+AI(K,K)*2
    IF (I1) 750,752,750
750  IF (I1-SMALL) 751,28,28
751  SMALL=I1
    IZ=K
28  CONTINUE
  GO TO 1B
752  IZ=IKK
  IF (ISL) 753,30,30
  EXACT EIGENVALUE - (A-ALPHA) SINGULAR. FLAG=2000
30  ISL=1
  ICI=2000
  DO 974 I=1,MM
    XR(I)=0.0
974  XI(I)=0.0
753  YR(IZ)=1.0
    YI(IZ)=0.0
    JJ=IZ
    BIG=1.0
    IF (IZ-MM) 33,32,33
32  IZZ=2
    GO TO 95
33  IZZ=IZZ+1
    DO 31 I=IZZ,MM
      YR(I)=0.
      YI(I)=0.
31  YI(I)=0.
    IZZ=MM-IZZ+2
    IF (IZZ-1) 95,49,95
  C  BACKWARD SUBSTITUTION
40  IZZ=1
41  BIG=0.
95  DO 46 I=IZZ,MM
    II=MM-I+1
    KK=II+1
    SR=0.
    SI=0.
    IF (I-1) 42,44,42
42  DO 43 K=KK,MM
      SR=SR+AR(II,K)*YR(K)-AI(II,K)*YI(K)
43  SI=SI+AR(II,K)*YI(K)+AI(II,K)*YR(K)
44  T1=AR(II,II)*2+AI(II,II)*2
      YR(II)=(AR(II,II)*XR(II)-SR)+AI(II,II)*(XI(II)-SI))/T1
      YI(II)=(AR(II,II)*XI(II)-SI)-AI(II,II)*(XR(II)-SR))/T1
      AM=YR(II)*2+YI(II)*2
      IF (AM-BIG) 46,46,45
45  JJ=II
      BIG=AM
46  CONTINUE
  C  NORMALIZATION - X=NORMALIZED Y
49  DO 47 I=1,MM

```



```

      XR(I)=(YR(JJ)*YR(I)+YI(JJ)*YI(I))/8IG
47  XI(I)=(YR(JJ)*YI(I)-YI(JJ)*YR(I))/8IG
      XR(JJ)=1.0
      XI(JJ)=0.0
92  DO 601 I=1,N
      DO 601 J=1,N
      AR(I,J)=BR(I,J)
601  AI(I,J)=BI(I,J)
116  IF (ISL) 755,50.60
755  GO TO IC
C    ALPHA RAYLEIGH QUOTIENT ~ (AX,X)/(X,X)=ALPHA
50  ALR=0.
      ALI=0.
      SUM=0.0
55  DO 52 I=1,N
      YR(I)=0.
      YI(I)=0.
      DO 51 K=1,N
      YR(I)=YR(I)+AR(I,K)*XR(K)-AI(I,K)*XI(K)
51  YI(I)=YI(I)+AR(I,K)*XI(K)+AI(I,K)*XR(K)
      ALR=ALR+XR(I)*YR(I)+XI(I)*YI(I)
      ALI=ALI+XR(I)*YI(I)-XI(I)*YR(I)
52  SUM=SUM+XR(I)**2+XI(I)**2
      ALR=ALR/SUM
      ALI=ALI/SUM
      AM=ALR**2+ALI**2
      IF (IEG) 1000,83,82
82  PRINT 300,ITWO,IF,ALR,ALI
C    TEST SECOND ITERATION
83  IS=0.
      DO 53 I=1,N
      I1=YR(I)-ALR*XR(I)+ALI*XI(I)
      I2=YI(I)-ALR*X1(I)-ALI*XR(I)
53  IS=IS+I1**2+I2**2
93  IF (IS/SUM-EP2)60,60,301
301  IF (IJ-MIT2) 99,400,400
400  PRINT 401,I1
401  FORMAT (54H      INVERSE POWER METHOD NOT CONVERGED ON TRY NUMBER
115)
      IF (IT-3) 402,990,402
402  ALR=ALR+GBR
      ALI=ALI+GBI
      IT=IT+1
      GO TO 4
60  ISL=0
63  PRINT 64, N,ALR,ALI,ICT,IJ
      VALR(N)=ALR
      VALI(N)=ALI
64  FORMAT (15,15H TH EIGENVALUE= 2E18.8,53X,2I5)
      SUMR=SUMR+ALR
      SUMI=SUMI+ALI
      T1=PRDR*ALR-PRDI*ALI
      PRDI=PRDR*ALI+PRDI*ALR
      PRDR=T1
C    DEFLATION OF MATRIX
      IF (JJ-N) 61,65,61
C    PERMUTATION OPERATION
61  I1=XR(JJ)
      I2=XI(JJ)
      XR(JJ)=XR(N)
      XI(JJ)=XI(N)

```



```

X1(N)=I1
X1(N)=I2
DO 68 K=1,N
  I1=AR(JJ,K)
  I2=AI(JJ,K)
  AR(JJ,K)=AR(N,K)
  AI(JJ,K)=AI(N,K)
  AR(N,K)=I1
  AI(N,K)=I2
68 AI(N,K)=I2
DO 62 K=1,N
  I1=AR(K,JJ)
  I2=AI(K,JJ)
  AR(K,JJ)=AR(K,N)
  AI(K,JJ)=AI(K,N)
  AR(K,N)=I1
  AI(K,N)=I2
62 AI(K,N)=I2
C DEFLATION
65 NN=N
  NN=N-1
DO 66 I=1,N
  DO 66 J=1,N
    AR(I,J)=AR(I,J)-XR(I)*AR(N+1,J)-X1(I)*AI(N+1,J)
    AI(I,J)=AI(I,J)-XR(I)*AI(N+1,J)-X1(I)*AR(N+1,J)
66 DO 600 I=1,N
  DO 600 J=1,N
    BR(I,J)=AR(I,J)
    BI(I,J)=AI(I,J)
600 BR(I,J)=AR(I,J)
    BI(I,J)=AI(I,J)
C COMPUTE EIGENVECTOR AND/OR DETERMINANT AS REQUIRED
910 IF (IDET) 1000,527,700
527 IF (IVEC) 1000,525,700
700 DO 702 I=1,M
  DO 702 J=1,M
    AR(I,J)=CR(I,J)
    AI(I,J)=CI(I,J)
    IF (I-J) 702,701,702
701 AR(I,I)=AR(I,I)-ALH
    AI(I,I)=AI(I,I)-ALI
702 CONTINUE
  MM=M
  INIER=0
  ASSIGN 911 TO IA
  GO TO 535
911 ASSIGN 530 TO IA
  IF (IDET) 1000,914,520
912 PRINT 913,DETR,DETI
913 FORMAT (58X,12#DETERMINANT= 2E18.8)
  ZLAG=SQRIF(AR(1,1)**2+AI(1,1)**2)
  ZLIIT=ZLAG
  DO 923 I=2,M
    ZMAGT=SQRTF(AR(I,1)**2+AI(I,1)**2)
    IF (ZLAG-ZMAGT) 922,920,920
920 IF (ZLIIT - ZMAGT) 923,923,921
921 ZLIIT=ZMAGT
  GO TO 923
922 ZLAG=ZMAGT
923 CONTINUE
  PRINT 924,ZLAG,ZLIIT
9240FORMAT (70H LARGEST AND SMALLEST MAGNITUDES OF DIAGONAL ELEMENTS
10F TRI. MATRIX= 2E18.8/)
914 ISL=-1
  IF (IVEC) 1000,916,915
915 DO 703 I=1,M

```



```

0929 XR(I)=0.
0930 XI(I)=0.
0931 ASSIGN 753 TO IB
0932 ASSIGN 704 TO IC
0933 GO TO 530
0934 910 ASSIGN 525 TO IC
0935 GO TO 92
0936 704 PRINT 705, (XR(I),XI(I),I=1,M)
0937 DO 707 I=1,M
0938 VECR(I,NN)=XR(I)
0939 707 VECI(I,NN)=XI(I)
0940 705 FORMAT (30H ASSOCIATED EIGENVECTOR IS:(2E20.8))
0941 525 IF (N-1) 526,67,523
0942 67 ALR=AR(1,1)
0943 ALI=AI(1,1)
0944 SUMR=SUMR+ALR
0945 SUMI=SUMI+ALI
0946 T1=PRDR*ALR-PRDI*ALI
0947 PRDI=PRDR*ALI+PRDI*ALR
0948 PRDH=T1
0949 PRINI 320,ALR,ALI
0950 VALR(1)=ALR
0951 VALI(1)=ALI
0952 320 FORMAT (20H FINAL EIGENVALUE= 2E18.8)
0953 N=0
0954 NN=1
0955 GO TO 910
0956 1000 STOP
0957 820 FORMAT(5X6HALPHA= 2E20.8)
0958 526 PRINT 321,SUMR,SUMI,PRDH,PRDI
0959 3210FORMAT (21H SUM OF EIGENVALUES= 2E18.8,
0960 125H PRODUCT OF EIGENVALUES= 2E18.8//)
0961 990 CONTINUE
0962 END
0963 END
0964

```

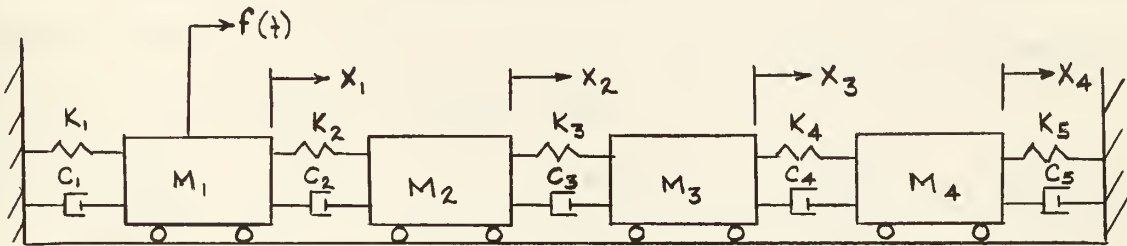




## APPENDIX E

### SAMPLE PROBLEM

The sample problem presented is that of a four degree of freedom system shown in the figure below. In order to show a comparison of data for damped and undamped systems, the example chosen is one in which the natural frequencies of the undamped system may be obtained with relative ease. The program is not restricted to problems with the symmetry displayed in the example.



Let  $M_1 = M_2 = M_3 = M_4 = 10 \text{ lb sec}^2/\text{in}$

$$K_1 = K_2 = K_3 = K_4 = 1000 \text{ lb/in}$$

$$C_1 = C_2 = C_3 = C_4 = 30 \text{ lb sec/in}$$

For the undamped case the frequency determinate then becomes

$$\begin{vmatrix} 2K - M\omega^2 & -K & 0 & 0 \\ -K & 2K - M\omega^2 & -K & 0 \\ 0 & -K & 2K - M\omega^2 & -K \\ 0 & 0 & -K & 2K - M\omega^2 \end{vmatrix}$$



Expanding the determinate

$$\omega^8 - 8\left(\frac{K}{M}\right)\omega^6 + 21\left(\frac{K}{M}\right)^2\omega^4 - 20\left(\frac{K}{M}\right)^3\omega^2 + 5\left(\frac{K}{M}\right)^4 = 0$$

The roots of the frequency equation are:

$$\omega_1 = 6.18034, \omega_2 = 11.75571, \omega_3 = 16.18034, \omega_4 = 19.02113$$

The problem was programmed with a forcing function  $f(t) = 100\sin(60t)$ , and an initial displacement on  $M_4$  of 0.5 inches. A step size of 0.005 seconds was used for the graphical option. All five options were called for and the resulting output is shown on the following pages.

In addition to the output presented for  $C = 30$  lbsec/in runs were made with  $C = 0.01, 0.10$ , and  $1.0$  lb sec/in. The frequencies and damping ratios for all runs are tabulated below.

| <u>C lb sec/in</u> | <u><math>\xi_1</math></u> | <u><math>\omega_1</math>, rad/sec</u> | <u><math>\xi_2</math></u> | <u><math>\omega_2</math>, rad/sec</u> |
|--------------------|---------------------------|---------------------------------------|---------------------------|---------------------------------------|
| 0.01               | —                         | 6.18034                               | 0.0001                    | 11.75571                              |
| 0.10               | 0.0003                    | 6.18034                               | 0.0006                    | 11.75570                              |
| 1.00               | 0.0031                    | 6.18028                               | 0.0067                    | 11.75530                              |
| 30.00              | 0.0098                    | 6.12699                               | 0.1821                    | 11.38430                              |

| <u>C lb sec/in</u> | <u><math>\xi_3</math></u> | <u><math>\omega_3</math>, rad/sec</u> | <u><math>\xi_4</math></u> | <u><math>\omega_4</math>, rad/sec</u> |
|--------------------|---------------------------|---------------------------------------|---------------------------|---------------------------------------|
| 0.01               | 0.0001                    | 16.18034                              | 0.0001                    | 19.02113                              |
| 0.10               | 0.0008                    | 16.18033                              | 0.0009                    | 19.02111                              |
| 1.00               | 0.0081                    | 16.17928                              | 0.0095                    | 19.01941                              |
| 30.00              | 0.2584                    | 15.19737                              | 0.3118                    | 17.40396                              |



## MATRIX ANALYSIS OF A MULTI-DEGREE OF FREEDOM VIBRATION SYSTEM WITH VISCOUS DAMPING.

SYSTEM OF ORDER 4

## INERTIA MATRIX

|          |          |          |
|----------|----------|----------|
| .100E+02 | .000E+00 | .000E+00 |
| .000E+00 | .100E+02 | .000E+00 |
| .000E+00 | .000E+00 | .100E+02 |
| .000E+00 | .000E+00 | .100E+02 |

## DAMPING MATRIX

|           |           |           |
|-----------|-----------|-----------|
| .600E+02  | .000E+00  | .000E+00  |
| -.300E+02 | -.300E+02 | .000E+00  |
| .000E+00  | .600E+02  | -.300E+02 |
| .000E+00  | .000E+00  | .600E+02  |

## STIFFNESS MATRIX

|           |          |           |
|-----------|----------|-----------|
| .200E+04  | .000E+00 | .000E+00  |
| -.100E+04 | .200E+04 | .000E+00  |
| .000E+00  | .200E+04 | -.100E+04 |
| .000E+00  | .000E+00 | .200E+04  |

THE INITIAL DISPLACEMENTS ARE

|          |          |          |
|----------|----------|----------|
| .000E+00 | .000E+00 | .500E+00 |
|----------|----------|----------|

THE INITIAL CONDITIONS OF VELOCITY ARE

|          |          |          |
|----------|----------|----------|
| .000E+00 | .000E+00 | .000E+00 |
|----------|----------|----------|

THE REAL PART OF THE AMPLITUDE OF THE DRIVING FORCE

|          |          |          |
|----------|----------|----------|
| .100E+03 | .000E+00 | .000E+00 |
|----------|----------|----------|

THE IMAG PART OF THE AMPLITUDE OF THE DRIVING FORCE

|          |          |          |
|----------|----------|----------|
| .000E+00 | .000E+00 | .000E+00 |
|----------|----------|----------|

THE FREQUENCY OF THE FORCING FUNCTION IS 60.0000RADIANS PER SECOND

THE STEP SIZE FOR THE GRAPHICAL OPTION IS .0050SECONDS

END OF INPUT DATA



|                           |                |                |                        |               |               |    |    |
|---------------------------|----------------|----------------|------------------------|---------------|---------------|----|----|
| TRACE OF MATRIX=          | -.24000000E+02 | .00000000E+00  | DETERMINANT OF MATRIX= | .50000000E+09 | .00000000E+00 | 20 | 27 |
| 8 IH EIGENVALUE=          | -.39270510E+01 | -.15698550E+02 |                        |               |               |    |    |
| ASSOCIATED EIGENVECTOR IS |                |                |                        |               |               |    |    |
| - .15000000E-01           | .59555484E-01  |                |                        |               |               |    |    |
| .92705098E-02             | -.37054527E-01 |                |                        |               |               |    |    |
| .92705098E-02             | -.37054527E-01 |                |                        |               |               |    |    |
| -.15000000E-01            | .59555484E-01  |                |                        |               |               |    |    |
| .10000000E+01             | .00000000E+00  |                |                        |               |               |    |    |
| -.61803399E+00            | -.12372172E-10 |                |                        |               |               |    |    |
| .10000000E+01             | .54640296E-11  |                |                        |               |               |    |    |
| 7 IH EIGENVALUE=          | -.54270510E+01 | -.18230483E+02 |                        |               |               |    |    |
| ASSOCIATED EIGENVECTOR IS |                |                |                        |               |               |    |    |
| -.92705098E-02            | .31141383E-01  |                |                        |               |               |    |    |
| .15000000E-01             | -.50387816E-01 |                |                        |               |               |    |    |
| -.15000000E-01            | .50387816E-01  |                |                        |               |               |    |    |
| .92705098E-02             | -.31141383E-01 |                |                        |               |               |    |    |
| .61803399E+00             | -.10998085E-09 |                |                        |               |               |    |    |
| -.10000000E+01            | .10173985E-09  |                |                        |               |               |    |    |
| .10000000E+01             | .00000000E+00  |                |                        |               |               |    |    |
| -.61803399E+00            | -.26054705E-10 |                |                        |               |               |    |    |
| 6 IH EIGENVALUE=          | -.54270510E+01 | .18230483E+02  |                        |               |               | 20 | 23 |
| ASSOCIATED EIGENVECTOR IS |                |                |                        |               |               |    |    |
| -.92705098E-02            | -.31141383E-01 |                |                        |               |               |    |    |
| .15000000E-01             | .50387816E-01  |                |                        |               |               |    |    |
| -.15000000E-01            | -.50387816E-01 |                |                        |               |               |    |    |
| .92705098E-02             | .31141383E-01  |                |                        |               |               |    |    |
| .61803399E+00             | .61674151E-09  |                |                        |               |               |    |    |
| -.10000000E+01            | -.44087270E-09 |                |                        |               |               |    |    |
| .10000000E+01             | .00000000E+00  |                |                        |               |               |    |    |
| -.61803399E+00            | .11290372E-09  |                |                        |               |               |    |    |
| 5 IH EIGENVALUE=          | -.39270510E+01 | .15698550E+02  |                        |               |               | 20 | 22 |
| ASSOCIATED EIGENVECTOR IS |                |                |                        |               |               |    |    |
| -.15000000E-01            | -.59555484E-01 |                |                        |               |               |    |    |
| .92705098E-02             | .37054527E-01  |                |                        |               |               |    |    |
| .92705098E-02             | .37054527E-01  |                |                        |               |               |    |    |
| -.15000000E-01            | -.59555484E-01 |                |                        |               |               |    |    |
| .10000000E+01             | -.19107862E-10 |                |                        |               |               |    |    |
| -.61803399E+00            | .71142697E-10  |                |                        |               |               |    |    |
| -.61803399E+00            | -.39790393E-10 |                |                        |               |               |    |    |
| .10000000E+01             | .00000000E+00  |                |                        |               |               |    |    |
| 4 IH EIGENVALUE=          | -.20729490E+01 | -.11571494E+02 |                        |               |               | 20 | 26 |
| ASSOCIATED EIGENVECTOR IS |                |                |                        |               |               |    |    |
| -.15000000E-01            | .83732120E-01  |                |                        |               |               |    |    |
| -.92705095E-02            | .51749296E-01  |                |                        |               |               |    |    |
| .92705098E-02             | -.51749296E-01 |                |                        |               |               |    |    |
| .15000000E-01             | -.83732120E-01 |                |                        |               |               |    |    |
| .10000000E+01             | .00000000E+00  |                |                        |               |               |    |    |
| .61803399E+00             | -.43108852E-08 |                |                        |               |               |    |    |
| -.61803399E+00            | .10941793E-08  |                |                        |               |               |    |    |
| -.10000000E+01            | .40973763E-08  |                |                        |               |               |    |    |
| 3 IH EIGENVALUE=          | -.20729490E+01 | .11571494E+02  |                        |               |               | 18 | 20 |
| ASSOCIATED EIGENVECTOR IS |                |                |                        |               |               |    |    |
| -.15000000E-01            | -.83732120E-01 |                |                        |               |               |    |    |
| -.92705100E-02            | -.51749296E-01 |                |                        |               |               |    |    |
| .92705099E-02             | .51749296E-01  |                |                        |               |               |    |    |
| .15000000E-01             | .83732120E-01  |                |                        |               |               |    |    |
| .10000000E+01             | .00000000E+00  |                |                        |               |               |    |    |
| .61803399E+00             | -.30441634E-08 |                |                        |               |               |    |    |
| -.61803399E+00            | .75569795E-09  |                |                        |               |               |    |    |
| -.10000000E+01            | .29058798E-08  |                |                        |               |               |    |    |

ITERATIONS REQ'D FOR CONVERGENCE - POWER METHOD  
 TOTAL ITERATIONS REQ'D FOR CONVERGENCE





2 1H EIGENVALUE= -.57294902E+00  
 ASSOCIATED EIGENVECTOR IS  
 -.92705098E-02  
 -.15000000E-01  
 -.15000000E-01  
 -.92705098E-02  
 .61803399E+00  
 .10000000E+01  
 .10000000E+01  
 .61803399E+00  
 .16258136E-09  
 .57294902E+00  
 FINAL EIGENVALUE= .61537249E+01  
 ASSOCIATED EIGENVECTOR IS  
 -.92705098E-02  
 -.15000000E-01  
 -.15000000E-01  
 -.92705098E-02  
 .61803399E+00  
 .10000000E+01  
 .10000000E+01  
 .61803399E+00  
 .16258136E-09  
 .57294902E+00

SUM OF EIGENVALUES= .24000000E+02  
 .58207661E-09  
 .50000000E+09  
 -.45410156E+00

THE NATURAL FREQUENCIES ARE

.61269944E+01 .11384303E+02

MODAL MATRIX, REAL PART

.61803399E+00 .10000000E+01  
 .10000000E+01 .61803399E+00  
 .10000000E+01 .61803399E+00  
 .61803399E+00 .10000000E+01

MODAL MATRIX, IMAG PART

-.21955839E-10 .00000000E+00  
 .00000000E+00 -.30441634E-08  
 .78542994E-10 .75569795E-09  
 .18759387E-09 .29058798E-08

THE COEFFICIENT MATRIX OF VEL X COS( 6.154T)

-.43624026E-11 -.57676395E-11  
 -.57676395E-11 -.72435552E-11  
 -.29136369E-11 -.26256819E-11  
 .32519677E-11 .65526704E-11

THE COEFFICIENT MATRIX OF X COS( 6.154T)

.13819660E+00 .22360680E+00  
 .22360680E+00 .36180340E+00  
 .22360680E+00 .36180340E+00  
 .13819660E+00 .22360680E+00



THE COEFFICIENT MATRIX OF VEL X SIN( 6.1541)  
 .22457390E-01 .36336820E-01 .36336820E-01  
 .36336820E-01 .58794210E-01 .58794210E-01  
 .36336820E-01 .58794210E-01 .58794210E-01  
 .22457390E-01 .36336820E-01 .36336820E-01  
 .22457390E-01 .36336820E-01 .36336820E-01

THE COEFFICIENT MATRIX OF X SIN( 6.1541)  
 .12866939E-01 .20819146E-01 .20819146E-01  
 .20819146E-01 .33686085E-01 .33686085E-01  
 .20819146E-01 .33686085E-01 .33686085E-01  
 .12866939E-01 .20819146E-01 .20819146E-01  
 .12866939E-01 .20819146E-01 .20819146E-01

THE COEFFICIENT MATRIX OF VEL X COS( 11.5711)  
 .13474983E-09 -.11901221E-10 -.59651733E-10  
 -.11901221E-10 -.66180573E-10 .21958416E-10  
 -.59651733E-10 .21958416E-10 .22263741E-10  
 -.43892319E-10 .68054292E-10 .34987015E-11  
 -.43892319E-10 .68054292E-10 .34987015E-11

THE COEFFICIENT MATRIX OF X COS( 11.5711)  
 .36180340E+00 .22360680E+00 -.22360680E+00  
 .22360680E+00 .13819660E+00 -.13819660E+00  
 -.22360680E+00 .13819660E+00 .13819660E+00  
 -.36180340E+00 -.22360680E+00 .22360680E+00  
 -.36180340E+00 -.22360680E+00 .22360680E+00

THE COEFFICIENT MATRIX OF VEL X SIN( 11.5711)  
 .31266782E-01 .19323934E-01 -.19323934E-01  
 .19323934E-01 .11942848E-01 -.11942848E-01  
 -.19323934E-01 .11942848E-01 .11942848E-01  
 -.31266782E-01 -.19323934E-01 .19323934E-01  
 -.31266782E-01 -.19323934E-01 .19323934E-01

THE COEFFICIENT MATRIX OF X SIN( 11.5711)  
 .64814446E-01 .40057530E-01 -.40057530E-01  
 .40057530E-01 .24756916E-01 -.24756916E-01  
 -.40057530E-01 .24756916E-01 .24756916E-01  
 -.64814446E-01 -.40057530E-01 .40057530E-01  
 -.64814446E-01 -.40057530E-01 .40057530E-01

THE COEFFICIENT MATRIX OF VEL X COS( 15.6971)  
 -.38605907E-12 .16062244E-11 -.95076869E-12  
 -.16062244E-11 -.18379411E-11 -.25763246E-12  
 -.95076869E-12 .13226762E-11 .13226762E-11  
 .54374643E-13 .13340214E-11 -.12229717E-11  
 .54374643E-13 .13340214E-11 -.12229717E-11

THE COEFFICIENT MATRIX OF X COS( 15.6971)  
 .36180340E+00 -.22360680E+00 -.22360680E+00  
 -.22360680E+00 .13819660E+00 .13819660E+00  
 -.22360680E+00 .13819660E+00 .13819660E+00  
 .36180340E+00 -.22360680E+00 .22360680E+00  
 .36180340E+00 -.22360680E+00 .22360680E+00

THE COEFFICIENT MATRIX OF VEL X SIN( 15.6971)  
 .23049868E-01 -.14245602E-01 -.14245602E-01  
 .23049868E-01 .88042662E-02 .88042662E-02  
 -.14245602E-01 .88042662E-02 .88042662E-02  
 .23049868E-01 -.14245602E-01 .14245602E-01  
 .23049868E-01 -.14245602E-01 .14245602E-01



THE COEFFICIENT MATRIX OF X SIN( 15.6971)

|                |                |                |                |
|----------------|----------------|----------------|----------------|
| .90518007E-01  | -.55943205E-01 | -.55943205E-01 | .90518007E-01  |
| -.55943205E-01 | .34574802E-01  | .34574802E-01  | -.55943205E-01 |
| -.55943205E-01 | .34574802E-01  | .34574802E-01  | -.55943205E-01 |
| .90518007E-01  | -.55943205E-01 | -.55943205E-01 | .90518007E-01  |

THE COEFFICIENT MATRIX OF VEL X COS( 16.230T)

|                |                |                |                |
|----------------|----------------|----------------|----------------|
| .10813906E-10  | -.10664918E-10 | .52573745E-11  | -.18644106E-11 |
| -.10664918E-10 | .62012254E-11  | .25483639E-11  | -.38156704E-11 |
| .52573745E-11  | .25483639E-11  | -.11297953E-10 | .92232140E-11  |
| -.18644106E-11 | -.38156704E-11 | .92232140E-11  | -.70850853E-11 |

THE COEFFICIENT MATRIX OF X COS( 18.230T)

|                |                |                |                |
|----------------|----------------|----------------|----------------|
| .13819660E+00  | -.22360680E+00 | .22360680E+00  | -.13819660E+00 |
| -.22360680E+00 | .36180340E+00  | -.36180340E+00 | .22360680E+00  |
| .22360680E+00  | -.36180340E+00 | .36180340E+00  | -.22360680E+00 |
| -.13819660E+00 | .22360680E+00  | -.22360680E+00 | .13819660E+00  |

THE COEFFICIENT MATRIX OF VEL X SIN( 18.230T)

|                |                |                |                |
|----------------|----------------|----------------|----------------|
| .75805232E-02  | -.12265544E-01 | .12265544E-01  | -.75805232E-02 |
| -.12265544E-01 | .19846067E-01  | -.19846067E-01 | .12265544E-01  |
| .12265544E-01  | -.19846067E-01 | .19846067E-01  | -.12265544E-01 |
| -.75805232E-02 | .12265544E-01  | -.12265544E-01 | .75805232E-02  |

THE COEFFICIENT MATRIX OF X SIN( 18.230T)

|                |                |                |                |
|----------------|----------------|----------------|----------------|
| .41139886E-01  | -.66565734E-01 | .66565734E-01  | -.41139886E-01 |
| -.66565734E-01 | .10770562E+00  | -.10770562E+00 | .66565734E-01  |
| .66565734E-01  | -.10770562E+00 | .10770562E+00  | -.66565734E-01 |
| -.41139886E-01 | .66565734E-01  | -.66565734E-01 | .41139886E-01  |

FREE VIBRATION OPTION NO. 1 HAS BEEN EXECUTED



THE COEFFICIENT COLUMN FOR EXP( -.573T)COS( 6.154T) .6910E-01 .1118E+00 .6910E-01

THE COEFFICIENT COLUMN FOR EXP( -.573T)SIN( 6.154T) .6433E-02 .1041E-01 .6433E-02

THE COEFFICIENT COLUMN FOR EXP( -2.073T)COS( 11.571T) -.1809E+00 -.1118E+00 .1809E+00

THE COEFFICIENT COLUMN FOR EXP( -2.073T)SIN( 11.571T) -.3241E-01 -.2003E-01 .3241E-01

THE COEFFICIENT COLUMN FOR EXP( -3.927T)COS( 15.697T) .1809E+00 -.1118E+00 .1809E+00

THE COEFFICIENT COLUMN FOR EXP( -3.927T)SIN( 15.697T) .4526E-01 -.2797E-01 .4526E-01

THE COEFFICIENT COLUMN FOR EXP( -5.427T)COS( 18.230T) -.6910E-01 .1118E+00 -.1118E-01

THE COEFFICIENT COLUMN FOR EXP( -5.427T)SIN( 18.230T) -.2057E-01 .3328E-01 .2057E-01

FREE VIBRATION OPTION NO. 2 HAS BEEN EXECUTED

GRAPH TITLED . . MIKLOS, T.J. JOB BOX M  
TRANSIENT PLUS STEADY STATE . . HAS BEEN PLOTTED.

GRAPH TITLED . . MIKLOS, T.J. JOB BOX M  
TRANSIENT PLUS STEADY STATE . . HAS BEEN PLOTTED.

GRAPH TITLED . . MIKLOS, T.J. JOB BOX M  
TRANSIENT PLUS STEADY STATE . . HAS BEEN PLOTTED.

GRAPH TITLED . . MIKLOS, T.J. JOB BOX M  
TRANSIENT PLUS STEADY STATE . . HAS BEEN PLOTTED.





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THE COEFFICIENT MATRIX OF THE CONVOLUTION INTEGRAL OF THE COS TERM FOR MODE 1 IS
-.43624026E-12
-.57676395E-12
-.7243552E-12
-.29136369E-12
.32519677E-12
.5526704E-12
.94066730E-12
.10866338E-11

THE COEFFICIENT MATRIX OF THE CONVOLUTION INTEGRAL OF THE SIN TERM FOR MODE 1 IS
.22457390E-02
.36336820E-02
.58794210E-02
.58794210E-02
.36336820E-02
.22457390E-02

THE COEFFICIENT MATRIX OF THE CONVOLUTION INTEGRAL OF THE COS TERM FOR MODE 2 IS
.13474983E-10
-.11901221E-11
-.60180573E-11
-.59651733E-11
.43892319E-11
.68054252E-11
.34987015E-12
.46965194E-11

THE COEFFICIENT MATRIX OF THE CONVOLUTION INTEGRAL OF THE SIN TERM FOR MODE 2 IS
.31266783E-02
.19323934E-02
.11942848E-02
.19323934E-02
.31266783E-02

THE COEFFICIENT MATRIX OF THE CONVOLUTION INTEGRAL OF THE COS TERM FOR MODE 3 IS
-.38605907E-13
.16062244E-12
-.18379411E-12
-.25763246E-13
.1326762E-12
.13340214E-12
.49480835E-13

THE COEFFICIENT MATRIX OF THE CONVOLUTION INTEGRAL OF THE SIN TERM FOR MODE 3 IS
.23049868E-02
-.14245602E-02
.88042662E-03
.88042662E-03
-.14245602E-02
.23049868E-02

THE COEFFICIENT MATRIX OF THE CONVOLUTION INTEGRAL OF THE COS TERM FOR MODE 4 IS
.10813906E-11
-.10664918E-11
.62012254E-12
.25483639E-12
.52573745E-12
-.18644106E-12
.92232140E-12
-.70850853E-12

THE COEFFICIENT MATRIX OF THE CONVOLUTION INTEGRAL OF THE SIN TERM FOR MODE 4 IS
.75805232E-03
-.12265544E-02
.19846067E-02
.12265544E-02
-.75805232E-03

```

FORCED VIBRATION OPTION NO. 4 HAS BEEN EXECUTED



THE REAL PART OF THE INVERSE

|            |            |            |            |
|------------|------------|------------|------------|
| -.2900E-04 | .5028E-06  | .8121E-07  | -.5627E-08 |
| .5028E-06  | -.2892E-04 | .4971E-06  | .8121E-07  |
| .8121E-07  | .4971E-06  | -.2892E-04 | .5028E-06  |
| -.5627E-08 | .8121E-07  | .5028E-06  | -.2900E-04 |

THE IMAG PART OF THE INVERSE

|            |            |            |            |
|------------|------------|------------|------------|
| -.3147E-05 | .1679E-05  | -.6702E-07 | -.2924E-08 |
| .1679E-05  | -.3214E-05 | .1676E-05  | -.6702E-07 |
| -.6702E-07 | .1676E-05  | -.3214E-05 | .1679E-05  |
| -.2924E-08 | -.6702E-07 | .1679E-05  | -.3147E-05 |

THE REAL PART OF THE IDENTITY MATRIX

|           |            |           |            |
|-----------|------------|-----------|------------|
| .1000E+01 | .9095E-12  | .0000E+00 | -.7105E-14 |
| .9095E-12 | .1000E+01  | .9095E-12 | .1137E-12  |
| .1137E-12 | .6821E-12  | .1000E+01 | .6821E-12  |
| .1776E-14 | -.5684E-13 | .2274E-12 | .1000E+01  |

THE IMAG PART OF THE IDENTITY MATRIX

|            |            |            |           |
|------------|------------|------------|-----------|
| -.1819E-11 | .0000E+00  | .5684E-13  | .0000E+00 |
| -.1819E-11 | .1819E-11  | -.1819E-11 | .1137E-12 |
| -.9948E-13 | .2728E-11  | -.1819E-11 | .9095E-12 |
| .0000E+00  | -.1421E-13 | .0000E+00  | .0000E+00 |

THE COEFFICIENT COLUMN OF COS(60.000T)

|            |           |           |            |
|------------|-----------|-----------|------------|
| -.2900E-02 | .5028E-04 | .8121E-05 | -.5627E-06 |
|------------|-----------|-----------|------------|

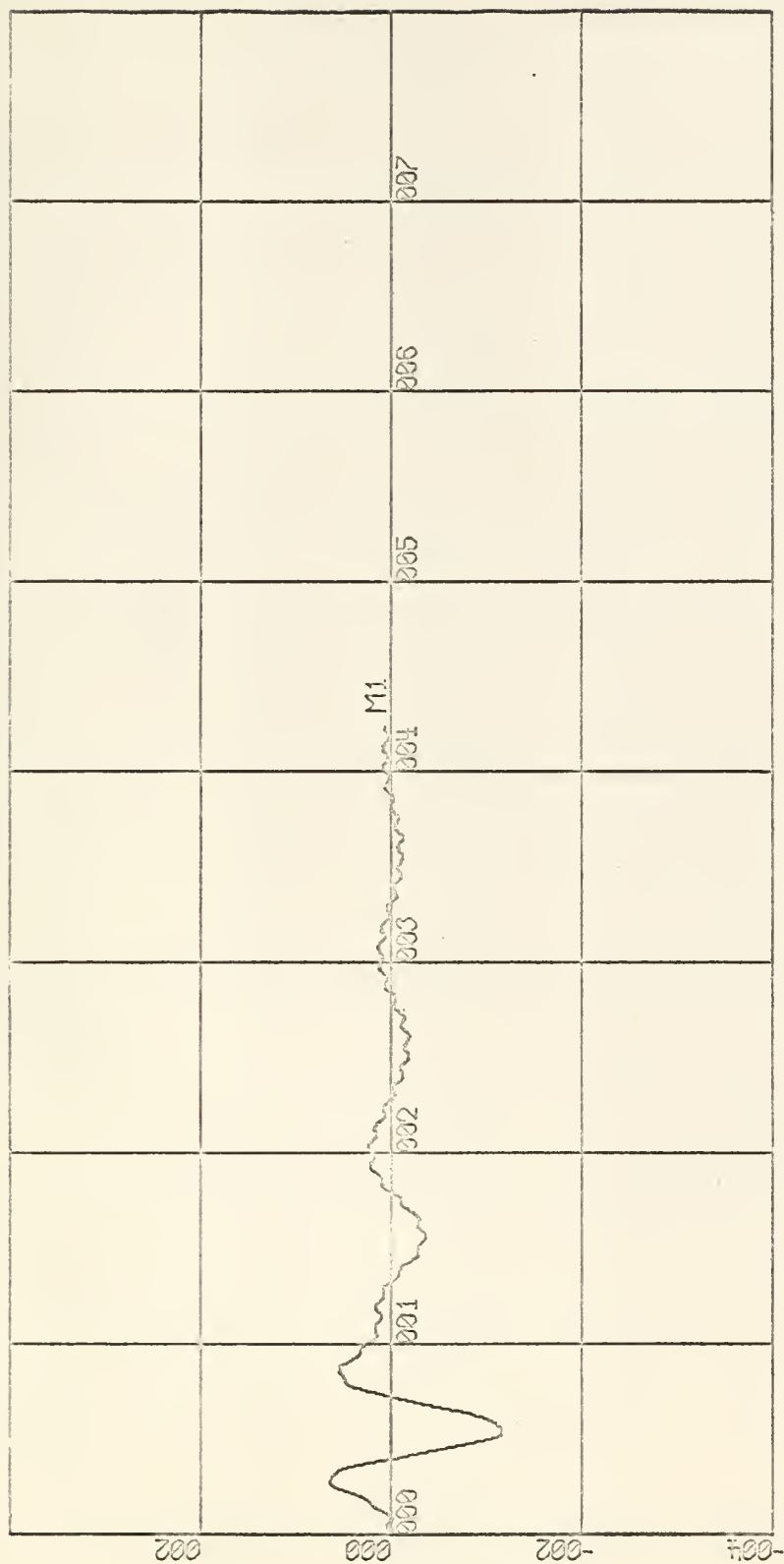
THE COEFFICIENT COLUMN OF SIN(60.000T)

|            |           |            |            |
|------------|-----------|------------|------------|
| -.3147E-03 | .1679E-03 | -.6702E-05 | -.2924E-06 |
|------------|-----------|------------|------------|

FORCED VIBRATION OPTION NO. 5 HAS BEEN EXECUTED

TIME, 6 MINUTES AND 55 SECONDS



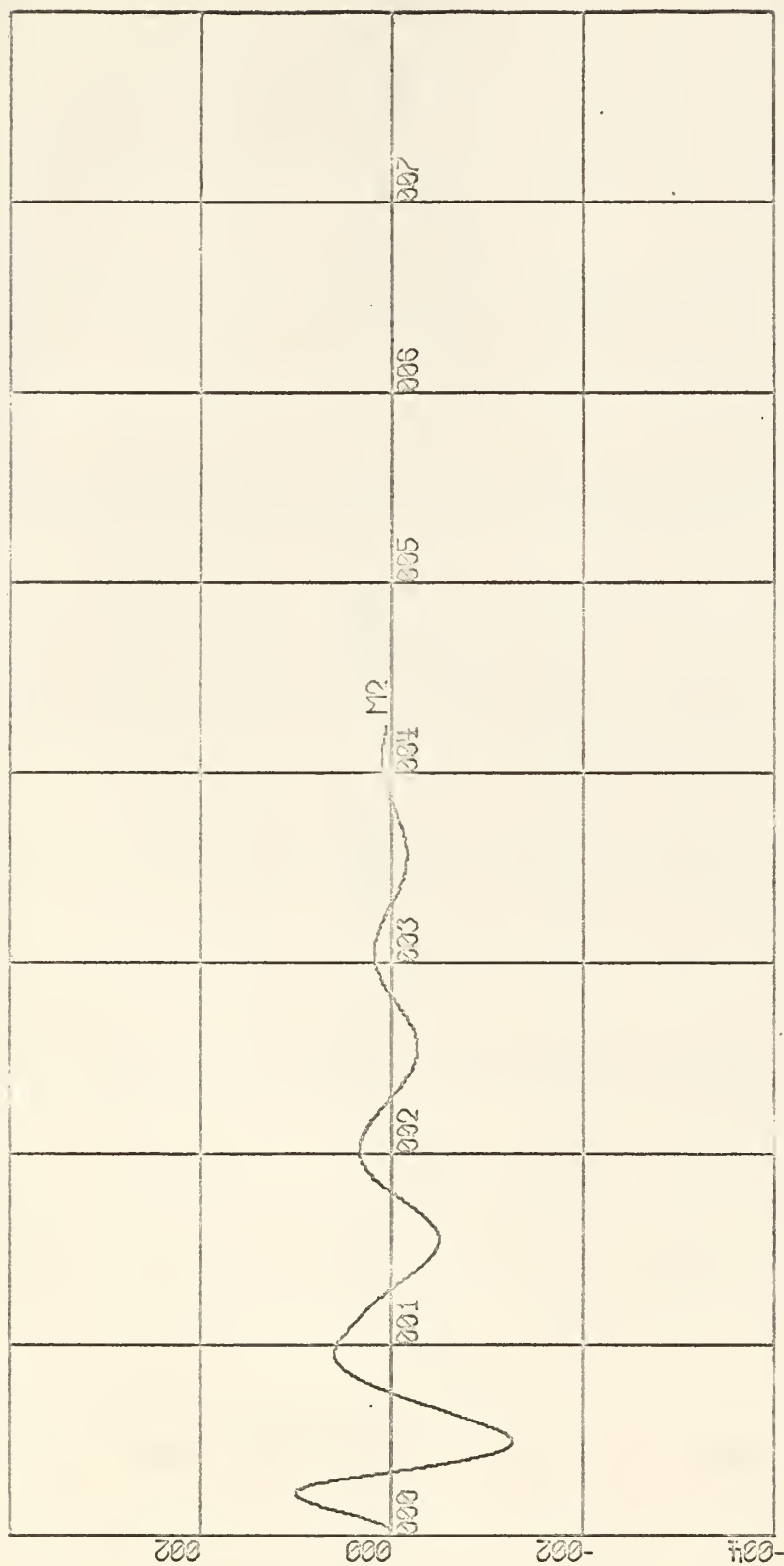


X-SCALE = 1.00E+00 UNITS/INCH

Y-SCALE = 2.00E-01 UNITS/INCH

MIKLOS, T.J.  
JOB BOX M  
TRANSIENT PLUS STEADY STATE





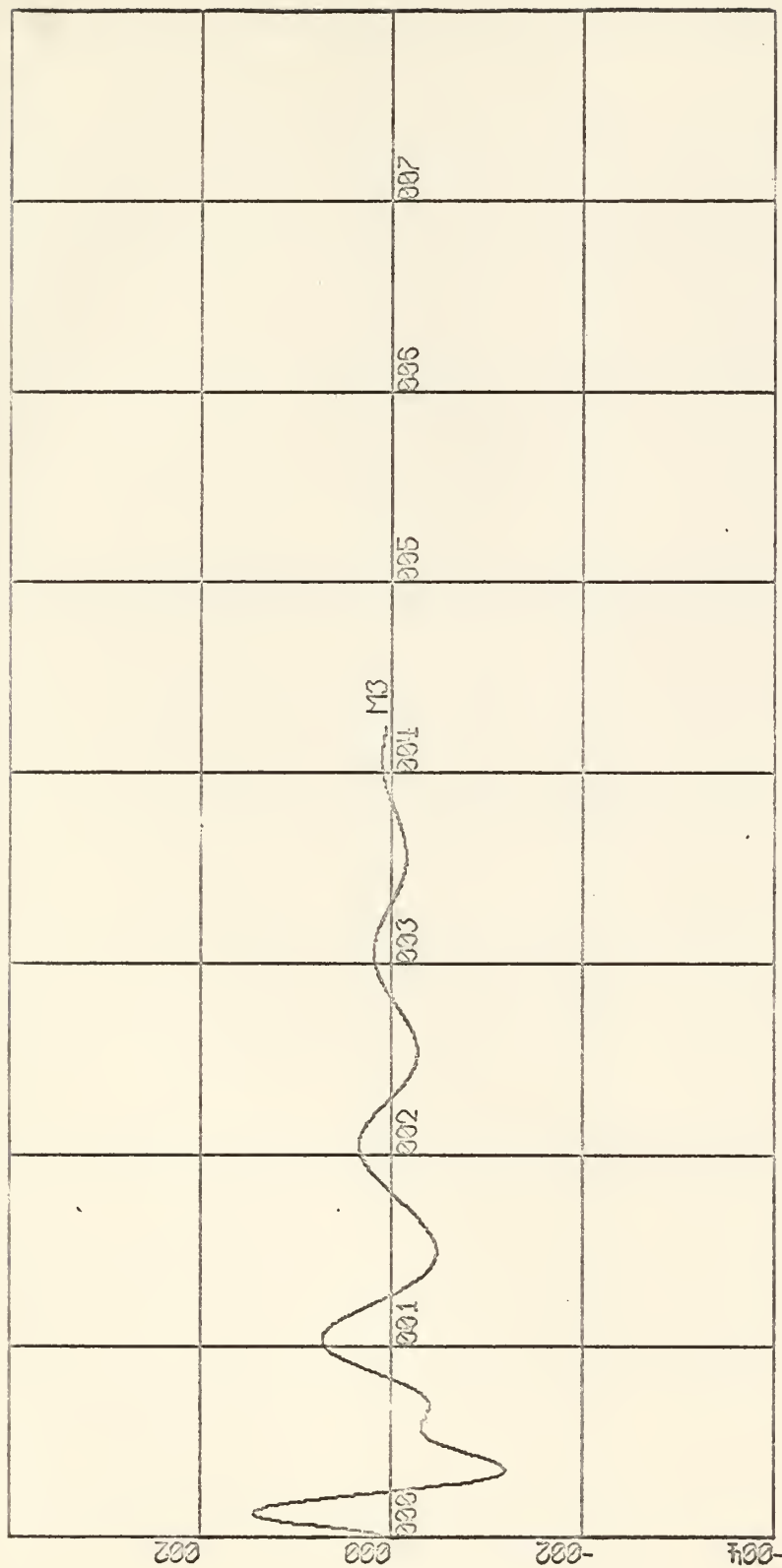
X-SCALE = 1.00E+00 UNITS/INCH.

Y-SCALE = 2.00E-01 UNITS/INCH.

MIKLOS, T.J. JOB BOX M  
TRANSIENT PLUS STEADY STATE





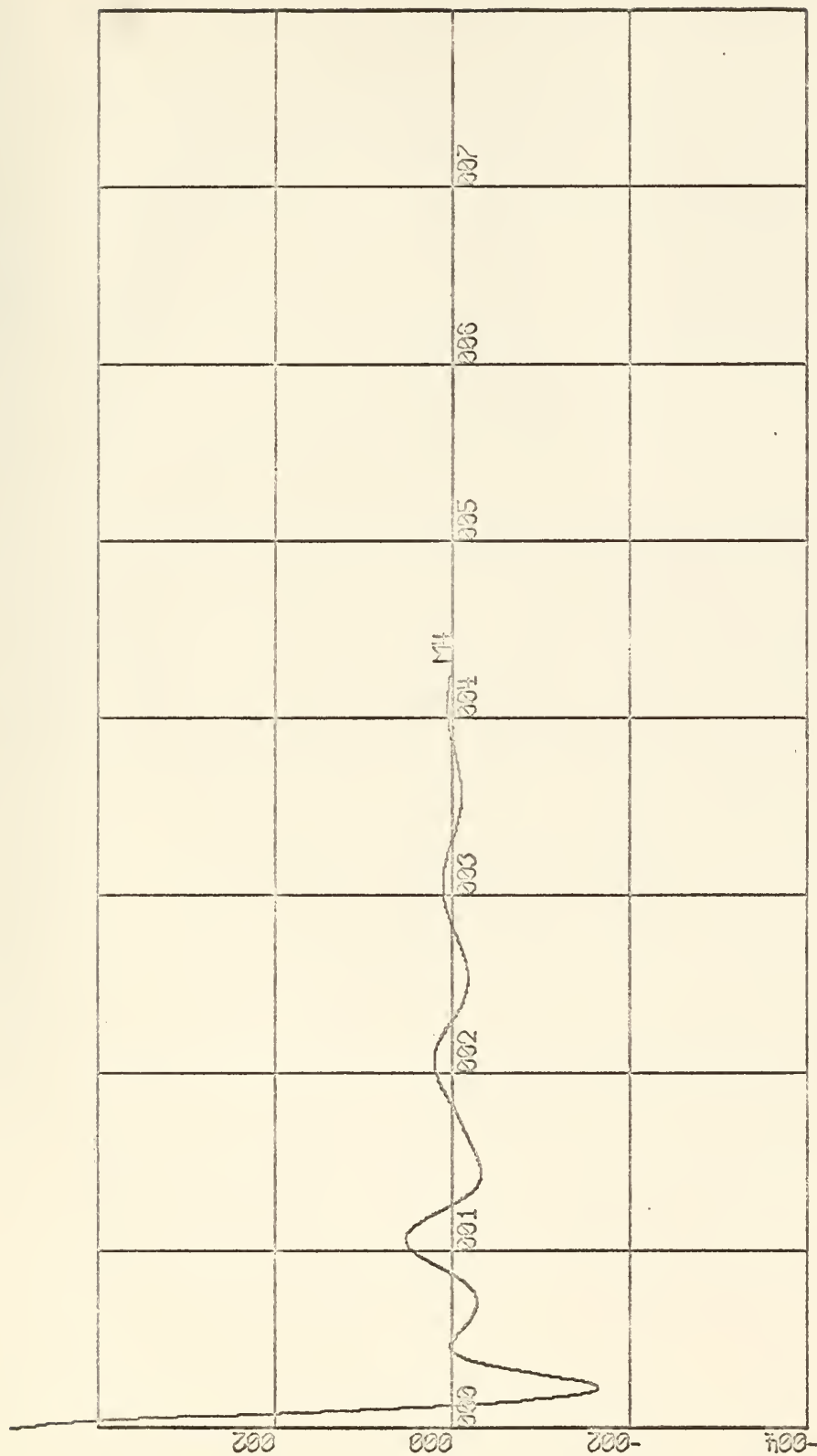


X-SCALE = 1.00E+00 UNITS/INCH.

Y-SCALE = 2.00E-01 UNITS/INCH.

MIKLOS, T. J. JOB BOX M  
TRANSIENT PLUS STEADY STATE





X-SCALE = 1.00E+00 UNITS/INCH.

Y-SCALE = 2.00E-01 UNITS/INCH.

MIKLOS, T.J. JOB BOX M  
TRANSIENT PLUS STEADY STATE















thesM5816

Analysis of a multi-degree of freedom vi



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